Affirmative Action in Higher Education: A Theoretical Perspective

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Abstract

Affirmative action refers to discrimination in favour of historically disadvantaged sections of society. On April 10, 2008, the Supreme Court of India upheld a controversial law that reserved 27% of seats in state universities for the Other Backward Classes (OBC), in addition to the 22.5% already reserved for the Scheduled Castes (SC) and Scheduled Tribes (ST). There has been little theoretical economic analysis of affirmative action in higher education in the Indian context, as most of the literature deals with US-style policies and institutional frameworks. We first build a baseline model with two castes that differ in their conditional distributions of human capital and where college admissions are determined by a mechanical cut-off rule. We find a decrease in inequality but also a fall in economy-wide human capital. We then allow schools to choose quality, and find that affirmative action results in a reduction in the quality of schools that educate high-caste students. Under certain conditions, schools educating low-caste students may also reduce quality, giving rise to the possibility of a general reduction in human capital without any beneficial effect on inequality. Lastly, we allow students to choose effort levels, and find that affirmative action induces more effort from a significant portion of the high-caste and less effort from a portion of the low-caste, again leading to the possibility of increased inequality.

JEL Classification: I21, I23, I28, I38, J7

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Chapter 1

Introduction

1.1 Affirmative Action in Higher Education

Discrimination in favour of historically disadvantaged sections of society is known by many names: 'affirmative action' in the US, 'positive discrimination' in the UK, or simply 'reservations' or 'quotas' in India and South Africa. The idea is one motivated by social justice ideals that place value on equality. It is argued that some sections of the population have been historically disadvantaged as a result of preference or taste-based discrimination, and even though this taste-based discrimination has now been outlawed, these sections are still socially and economically 'backward' when compared to the rest of the population. Examples include the African-Americans and other ethnic minorities in the US who were victims of first slavery and then racial discrimination, native Africans in South Africa who were victims of Apartheid, and the lower castes[1] in India who were victims of the rigid rules and prejudices arising from the caste system.

The objective of affirmative action is to 'undo' the discriminations of the past through reverse discrimination, i.e. discriminating in *favour* of these 'backward' groups so as to reduce the social and economic gap between them and the 'forward' sections of the population. One area where affirmative action policies are seen in abundance is the labour market, where affirmative action usually translates into preferential treatment towards applicants belonging to disadvantaged groups. In countries such as India, there are explicit quotas in the government sector for people belonging to disadvantaged groups. Thus, job allocation decisions are made based not just on grounds of merit, but also the social background of the candidate.

[1] A brief introduction to the caste system of pre-modern India will be provided in Section 1.3.

Education is another area where affirmative action policies are prevalent. Measures are usually concentrated at the higher education (i.e. university) level. Often, candidates with worse credentials belonging to a disadvantaged group are preferred for college admission over a candidate with better credentials belonging to an advantaged group. The argument is three-fold: firstly, the candidate from the disadvantaged group is more likely to have grown up in a disadvantaged environment, and so has not had the opportunities to excel that many candidates from advantaged groups enjoy; secondly, preferring the disadvantaged group candidate fulfils an obligation that society has to undo past wrongs done towards that group; and thirdly, diversity in college is a desirable end in itself[2] as a more diverse student community enhances the overall 'college experience'.

There has been a substantial amount of academic work on affirmative action, but most of it has been centered around the US. This thesis focusses on affirmative action in countries like India, where institutional structures, and therefore the required points and methods of enquiry are sometimes quite different.

1.2 Peculiarities in the Indian Education System

As is supported by the findings in Kingdon (1996), the free education provided by the state is, for the most part, considered to be of extremely low quality, and consequently a major chunk of educational services is provided by private, unaided schools. Whereas only the extremely wealthy send their children to elite private schools in the west, private schooling is more widespread in India, as evidenced by the increased demand for private unaided schools even in remote rural areas by people with very low income levels (Kingdon, 1996, p.75). The discernable quality and input gap between private and public schools has important implications for access to higher education given the large share of education catered to by the private sector.

Another major difference from the west is in college admissions. Universities in the west often consider applicants on a case by case basis. The candidate may be interviewed and several other factors apart from school grades are considered (pieces of written work, SAT scores in the US, etc.). In India, the applicant pool is so large that the sheer logistics of following such a system are incomprehensible. Instead, college admissions follow a mechanical 'cut-off'

[2]It is interesting, though, to note that this explanation came into fashion in the US after the Supreme Court struck down the use of minority quotas in admissions policies by rejecting the justification of remedying past injustices in *Regents of University of California vs. Blake (1978).*

rule. Students sit for nation-wide examinations in the last year of schooling[3] (conducted by one of several boards of secondary education) and obtain grades[4]. They then apply to the college they wish to go to, following which the college announces a 'cut-off', and students with grades equal to or greater than this threshold are admitted. The main point here is that while in the west, colleges can be seen as vital decision-making agents, it is incorrect to do so in the context of India as college admissions are, for all practical purposes, mechanical.[5]

Thus, institutional structures are very different in India when compared to the west, and these differences mean that the vast majority of the literature, which is based on western institutional frameworks, has limited relevance.

1.3 The Caste System and Affirmative Action in Indian Higher Education

The caste system, prevalent in large tracts of pre-modern India, postulated a societal hierarchy of four main castes: *Brahmins* (priests and teachers), *Kshatriyas* (warriors and aristocracy), *Vaishyas* (traders and merchants) and *Shudras* (the serving class). Each of these was further divided into several sub-castes. In addition, there were groups like the *Dalits* or 'untouchables' who were considered too low to be included in the caste system. Castes were originally associated with occupations and later became hereditary. The lower castes were discriminated against in that they were given the lowest paying and most undesirable jobs and as a result were socially and economically 'backward' in relation to the upper castes. Caste identification even in modern times is relatively simple, as members of a particular caste in a region are usually associated with a particular surname.

Although taste-based discrimination along caste lines has been outlawed in modern India, the lower castes still lag behind the upper castes. Affirmative action in India is an attempt to bring the lower castes up to speed with the rest of the population by means of quotas in higher education and government jobs. As is described in Bertrand, Hanna, and Mullainathan (2008), the Constitution since 1951 has allowed for 15% of seats in state universities to be

[3]In some cases, especially for engineering and medical courses, students sit for standardized examinations that are not part of the normal schooling process.

[4]Grades in India are actual numbers, e.g. marks obtained out of 100.

[5]This is not to say that colleges are not decision making agents. The cut-off strategy described here is a rational trigger strategy of a bayesian game in which colleges look to maximize the quality of their intake while observing only grades. However, due to the limited information observed by the colleges, their decision is rendered uninteresting from an analytical point of view. Therefore, in our analysis, we will treat college admissions as mechanical.

reserved for the Scheduled Castes (SC) and 7.5% for Scheduled Tribes (ST). These consist of historically the lowest castes, the *Dalits* and the *Adivasis* or tribal people. In its 2006-2007 winter session, the Indian Parliament passed a bill that provided for an additional reservation of 27% for the Other Backward Classes (OBC), a group of castes that were more backward than the upper castes, but less so than the SC or ST.[6] In April 2008, after initially giving a stay order, the Supreme Court upheld this law.[7] The time-frame for implementation was set to three years starting with the 2008-09 academic year. When fully implemented, total reservations for backward castes will stand at 49.5%.

The OBC reservations led to widespread polarization of opinion, with some supporting and others opposing the policy. There were several anti-reservations agitations[8] across the country in May 2006 whereas other sections of society and almost all political parties stood firm in support of the policies. Several of the issues touched upon in the ensuing public debate also find mention in Ghosh (2006) and *The furore over reservations: a primer, The Siliconeer, June 2006*. Some of the issues are as follows.

Firstly, is it morally correct to discriminate against someone merely because they were born to high-caste parents, something that is clearly outside their control?[9]

Secondly, whom does affirmative action actually benefit? In the modern situation where not all low-caste people are disadvantaged and not all high-caste people are advantaged, many believe that affirmative action actually benefits the 'creamy layer' among the low-castes, i.e. people belonging to lower castes who are socially and economically advantaged. The Supreme Court, in most of its rulings relating to affirmative action, has recommended that reservations should not be extended to the 'creamy layer'.[10]

Thirdly, are the lower castes able to take advantage of affirmative action? The argument is that the less meritorious lower caste candidates let in at the cost of more meritorious higher caste ones do not have the required skills to cope with the level of education, and are thus not able to take as much advantage of a college education as more meritorious students would

[6] The Mandal Commission report of 1978 identified 3747 castes as backward, which in the 1931 census formed 52% of the population (caste-based information has not been collected in any census since then). However, it only recommended 27% reservations due to a Supreme Court ruling that the government reservations could not exceed 50%. (Bertrand et al., 2008, p.6) Projections of the current OBC population vary. The National Sample Survey Organization (NSSO), based on the 61st round of the National Sample Survey (NSS) carried out in 2004-05 estimated a share of 40.94%. This information is available in *NSS Report No. 514*, which is available at the NSSO website (`http://mospi.nic.in/nsso_test1.htm`).

[7] See *SC allows 27% quota for OBCs, The Times of India, 11 April 2008.*

[8] See, for instance, *Delhi medical students plan indefinite fast, The Hindu, 14 May 2006.*

[9] See *Letters to the editor: Reservation issue, The Hindu, 1 May 2006.*

[10] See *SC allows 27% quota for OBCs, The Times of India, 11 April 2008.*

have done. There is thus a deterioration in education standards.[11]

Fourthly, should attention not be focussed at the school level, where the gap between private and government schools leads to a substantial portion of the population having a handicap even before entering college?[12]

Lastly, how effective are these policies in reducing inequality? As is mentioned in Bertrand et al. (2008), reservations were originally set to expire in 1960, but have since been extended many times. The current expiry is set for 2010, but it is likely to be extended again. Moreover, the list of castes receiving benefits has steadily grown over the years.

An issue that has surprisingly not been raised very often is the effect of affirmative action policies on the incentives for effort. Could affirmative action cause lower castes to 'take it easy', as they are assured of getting into college without working too hard, or does it motivate people to try harder, as extremely poor people can now harbour realistic aspirations of going to college? Answers to such questions could potentially have profound effects on the effectiveness of affirmative action policies.

1.4 Questions of Interest

A rigorous approach to all aspects of affirmative action requires inputs from not just economics, but also philosophy, law and political science. Even if we restrict ourselves to an economic analysis of the topic, an M.Phil. thesis can, at most, explore only a few facets of affirmative action.

The prime motivating factor behind this thesis is a lack of effort in the economic literature to rigorously analyze the complex incentive mechanisms that influence the outcomes of affirmative action policies under institutional structures different from those in the west. Specifically, there is a desire to provide a theoretical framework that better fits the circumstances in India.

That Indian institutional frameworks have been widely neglected in the economic literature relating to affirmative action is surprising, since India is home to more people than one billion people, which is more than the populations of the US and the EU combined. Surely, if over one-sixth of humanity is affected by these institutional structures, it deserves some attention from the economic fraternity?

[11]See, for example, *Quota fallout: Merit goes for a toss in GATE entrance, The Times of India, 23 May 2008.*

[12]See *Beyond the rhetoric of reservation, The Hindu, 28 May 2006.*

The specific questions of interest that we seek to answer in this thesis are as follows:

1. Does affirmative action reduce inequality?
2. What incentives does affirmative action in college admissions create for schools to invest in their students?
3. What incentives does affirmative action create for students' effort levels?

1.5 Structure of the Thesis

The thesis is structured as follows. In Chapter 2, we review some related literature. In Chapter 3, we build a simple baseline model where an individual can either belong to a high or a low-caste, and where historical taste-based discrimination is manifested in the fact that the high-caste's distribution of human capital (and hence earnings) stochastically dominates that of the low-caste. In doing so, we capture the idea of the low-caste being somehow disadvantaged in relation to the higher caste. We construct a mechanism whereby this initial difference leads to inequality in the human capital attainment of the next generation after education. We find in the simple model that affirmative action reduces inequality in human capital, but this reduction comes with a loss in aggregate human capital.

In Chapter 4, we extend the basic model by allowing schools to choose quality. We construct an equilibrium where high-caste students go to better quality schools than low-caste students, and in doing so we capture the notion that high-caste children are often given better opportunities than their low-caste counterparts. Here, we find that affirmative action leads to a reduction in quality for schools educating high-caste students, whereas the effect on the quality of schools educating low-caste students is ambiguous. We find that under certain circumstances all schools can experience a reduction in quality, and affirmative action may actually increase inequality in human capital.

Chapter 5 extends the basic model to allow students to choose their effort levels, and in doing so allows incentives for effort to be analyzed explicitly. We find that incentives for effort increase for some members of both castes and decrease for other members. Again, we find that under certain circumstances, it is possible for affirmative action to increase inequality.

Chapter 6 concludes.

Chapter 2

Literature Review

2.1 Theoretical Contributions

A seminal paper in the field of affirmative action is Coate and Loury (1993), which builds a model of the interaction between workers' decisions to invest in qualifications and employers' job-assignment decisions. Employers have prior beliefs about the average level of qualification of a group of workers, and they use this in conjunction with noisy test scores to set standards for the group, i.e. a test score above which workers are assigned to the more demanding task which can only be performed by qualified workers. This affects the workers' investment decision as it determines the probability of being assigned to a higher-paying task. In equilibrium, the proportion of workers that choose to invest in qualifications must tally exactly with the employers' beliefs. The existence of multiple equilibria and the possibility of different ex-ante identical groups of workers being stuck in different equilibria can lead to statistical discrimination. It is then shown that if the groups are ex-ante identical, under certain conditions affirmative action can break negative stereotypes about minorities stuck in a low-level equilibrium, but it is also possible to have a 'patronizing equilibrium', where standards are lowered for the minority group and raised for the other group to achieve the desired intake, leading to a widening of the qualifications gap. This paper established employers' beliefs as a crucial factor in affirmative action analysis. This is not very relevant to the current work as in our context colleges are not decision making agents, so beliefs do not enter into our analysis. Moreover, we model the two groups as ex-ante different due to the effects of taste-based discrimination in the past.

We now examine some contributions that relate to affirmative action in education. Many of these contributions are concerned with providing efficiency arguments for affirmative action.

Durlauf (2008) builds a simple model where a college student's human capital is determined by human capital accumulated during youth, human capital of other college students and college quality. It is then argued that an affirmative action admissions policy could be more efficient (in terms of maximizing aggregate human capital) than a meritocratic one if students perform better when interacting with students of different abilities (essentially a variant of the diversity argument) and if low human capital students are able to take better advantage of a high quality university education than high human capital students. It is also noted, however, that there has been very little empirical work to make informed conjectures about whether these conditions hold in practice. In this thesis, we abstract from the diversity argument in that we ignore peer effects in education. This is primarily because, firstly, diversity seems to be used as a proxy for the social justice argument, and secondly it is unclear as to whether and how diversity actually influences human capital. Also, we present some empirical evidence from Bertrand et al. (2008) that suggests that students from disadvantaged backgrounds experience a lower absolute increase in human capital from a college education.

De Fraja (2002) builds a model with several groups, with each group's distribution of ability first order stochastically dominating the last. Income depends on ability, own education and the general education level in the economy (which is an externality). In the absence of government intervention, private maximization exercises to determine investment in education do not take into account the externality. When the government intervenes and offers public education, it does so, and would like to give the high ability people more education. But ability is not observable, and in order to induce truthful revelation the government must drastically reduce education provided to the low ability people, who then find it better to revert to private education. Because, firstly, the government has limited funds to subsidize education, and secondly, disadvantaged groups have fewer high ability individuals (due to the crucial assumption that the hazard rate is higher for advantaged groups), the most efficient outcome attainable under asymmetric information is one where disadvantaged group members receive more education than advantaged group members of equal ability. Here, the argument for any intervention relies on the presence of an externality, which we wish to de-emphasize in our modeling.

Rotthoff (2008) builds a model where colleges care about placing their students in employment. It examines firms' hiring decisions, and argues that hiring a candidate belonging to a protected community entails an extra cost, i.e. the possibility of lawsuits particular to the protected community, like failing to hire on the basis of race. Since hiring based lawsuits, which are more common, tend to incentivize the hiring of protected community workers, firms will give preference to protected group applicants. As colleges care about placement, they will also want to give preference to individuals from the protected group. This work, again, relies on colleges acting as agents during the admissions process, whereas in the situation we wish to model admissions are mechanical.

Rey and Racionero (2008) also rely on an externality to justify affirmative action. Their model is one of inter-generational interactions. Education is costlier for people of uneducated parents, and this effect is larger for the minority group. Private decisions about whether to invest in education is inefficient because agents fail to account for the fact that their being educated increases the chances of their children being educated. Since this externality is larger for the minority group, they also experience more underinvestment in education. Again, the result is a direct consequence of an externality, which we abstract from.

Epple, Romano, and Sieg (2008) build a rich model of education provision with colleges that differ in initial endowments, where college quality explicitly depends positively on racial diversity. Colleges choose, among others, an admissions rule to maximize quality. In equilibrium, tuition is set equal to the reservation price and colleges will admit a student whose reservation price is not less than their effective marginal cost. Due to the presence of racial diversity in the quality index function and the assumption that the minority class is under-represented in college, the effective marginal cost for a minority applicant is lower as admitting him enhances racial diversity. As a result, minority students pay lower tuition fee and attend higher quality schools than their equally qualified non-minority peers. If affirmative action is disallowed, the paper shows that colleges will try to achieve the diversity objective by exploiting the correlation of ability and parental income with race. The model is then calibrated assuming Cobb-Douglas functional forms and key parameters are chosen based on survey data. They predict that proscription of affirmative action would result in a 35% decline in minority representation in the top tier colleges. The results of this paper hinge on the colleges wanting diversity, which the authors admit is a controversial claim.

Fryer, Loury, and Yuret (2008) model two groups differentiated by the costs of effort, with members of the disadvantaged group more likely to have higher costs than their advantaged group counterparts. Colleges (or firms) must choose an acceptance policy. It is shown, analogous to above, that if colleges seek racial diversity while not being allowed to discriminate based on race, they will give increased weightage to (imperfect) proxies that are correlated with race in their acceptance policies. Another important result in this paper concerns the effect of affirmative action on effort incentives. It is shown that if the marginal applicant is qualified (i.e. chooses to exert effort), then pursuit of affirmative action that seeks a minor increase in representation of the under-represented race leads to a reduction in the effort incentives of the advantaged group and an increase for the disadvantaged group when compared to the case where colleges do not care about diversity. If the marginal applicant is not qualified, then the results of such affirmative action are reversed. If, however, a major increase in representation is sought and the marginal applicant is at the border of being or not being qualified, then the qualifications of both groups will decrease. In Chapter 5, we explore the effect of affirmative action on effort incentives under different modeling specifications.

2.2 Empirical Contributions

As we are primarily concerned with affirmative action in higher education in the context of conditions prevailing in countries such as India, the empirical literature focussing on the US has limited appeal. We briefly review one such contribution. Howell (2004) builds a structural model of students' application decisions and colleges' acceptance decisions. The structural parameters are estimated by maximum likelihood estimation using National Education Longitudinal Survey (NELS) data which surveyed a cohort of students from 1988 to 2000. Based on this model, Howell predicts that banning affirmative action in the US would not significantly affect disadvantaged group representation, except in the most selective four-year institutions, where representation would fall by 3.3%. The author prescribes the replacement of race-based affirmative action in the US with improved support programmes for minority groups.

There has been surprisingly little research carried out in the Indian context. The only major works that the author is aware of are two recent contributions, inspired perhaps by the recent public debate in India following the introduction of the OBC quota. Desai and Kulkarni (2008) build a logit model of the probability of transitioning from one education level to the next using National Sample Survey (NSS) data between 1983 and 2000. They define an educational gap as a difference in the probability of proceeding to the next level of education conditional on having completed the previous level. They find that the educational gap between Hindus and Muslims (who are not a target group of the affirmative action program) has continued throughout the period of affirmative action, whereas the gap between upper caste groups and *Dalits* (who are a target group) has decreased. However, the decline has been at the school level whereas affirmative action has been focussed at the college level. The authors propose that the reduction in the educational gap is due more to affirmative action in employment. They do not find a strong relationship between affirmative action and a reduction in inequality. As an aside, their data contains no evidence that benefits are cornered by the 'creamy layer' of the lower caste groups.

By far the most closely related work to this thesis is Bertrand et al. (2008). The authors collected data on several individuals applying to engineering colleges in 1996 in one Indian state. They then use this data to compare the profiles of the backward-caste candidates benefiting from affirmative action to the other higher-caste candidates. They primarily compare socio-economic backgrounds, the increase in earnings due to a college education and labour market outcomes. Several of these findings may be viewed as stylized facts which are used at times to justify modelling assumptions. Also, the models presented behave in ways that are largely consistent with these findings. The main findings that relate to this thesis are as follows. Firstly, cut-off scores for high-caste applicants were higher than for low-caste applicants. Secondly, this study found that affirmative action successfully targets the economically disadvantaged at the margin, in that the mean parental income of upper-caste candidates who lost seats due to affirmative action was higher than that of those lower-caste applicants who gained seats. However, they also suggest that among the lower castes, it is the economically better off that benefit more. Thirdly, an engineering college education increases lower-caste members' income by between 40% and 70%, which is indistinguishable from the rate of increase enjoyed by the higher-caste members. Fourthly, there is an absolute cost involved with these programs as engineering college raises the income of high-caste students by more in absolute value than the increase enjoyed by the lower-caste students. Fifthly, higher-caste applicants are more likely to have been educated at English-medium private schools. Lastly, there is slight evidence for a reduction in inequality overall, but "large standard deviations prevent. . . unequivocal conclusions"(Bertrand et al., 2008, p.18).

Chapter 3

The Baseline Model

We shall first build a Donald Duck[1] model in order to obtain baseline results. As one of our questions of interest is whether affirmative action reduces inequality, we must specify a measure of inequality. To this end, we focus on average realized human capital, and we define inequality as the difference in the average realized human capital between castes. We also look at the effect of affirmative action on economy-wide average human capital. Our outcome measures are, thus, average realized human capital for each caste, inequality in human capital and economy-wide average human capital.

3.1 Model Set Up

Let there be a population of n people. A fraction λ are 'high-caste' (denoted H) and a fraction $(1-\lambda)$ are of 'low-caste' (denoted L). These individuals each undergo schooling and obtain grades. Based on these grades, they compete for $\bar{A} < n$ college seats. College admissions are mechanical, in the sense that the $\bar{A}$ people with the highest grades are admitted (we call this a 'cut-off' admissions policy). Then those that are admitted to college undergo higher education, after which payoffs are received. In this model, the payoff to each of the n people in the population is the level of human capital they end up with after education.

The specification of this very simple model means that there are no decisions taken by any economic agents. The whole process is completely mechanical, and so deriving results does not involve solving for optimal behaviour of any kind.

[1] In the author's view, a particular Disney character has had an unfair hegemony when it comes to the naming of baseline models. This is an attempt at propagating the cause of the other Disney (and indeed, non-Disney) characters that have been so sadly sidelined by academics the world over.

3.1.1 School Education

School grades g are a function of school quality $q \geq 0$, the student's ability $a \geq 0$, parents' human capital $p \geq 0$ and effort choice $e \geq 0$.

$$g = g(q, a, p, e) \tag{3.1}$$

It is fairly obvious as to why a, e and q should enter the grades function. The fact that p enters captures the fact parents with high human capital will positively affect their child's school attainment by providing better home teaching, and/or (as high human capital generally translates into higher income) by hiring better home tutors and providing better infrastructural resources such as better books.

We now make some plausible assumptions on g.

Assumption 3.1. *For simplicity, $g(q, a, p, e)$ is continuous and twice differentiable. Also, (i) g is increasing in all its arguments; (ii) all second order cross derivatives are positive; and (iii) $g(0, a, p, e) = g(q, 0, p, e) = g(q, a, 0, e) = g(q, a, p, 0) = 0$*

(i) ensures that higher ability students end up with higher grades, and so on. (ii) means that educational inputs are complementary.[2] (iii) is a simple level fixing assumption made for convenience.

To derive baseline results, we make the following simplifying assumption.

Assumption 3.2. *q is non-stochastic and constant across all schools and e is non-stochastic and constant across the entire population.*

We shall vary q in Chapter 4 and e in Chapter 5.

3.1.2 College Education

School grades represent human capital attained at the end of school. For those who go to college, human capital is further enhanced as follows.

$$k = k(\mu, g) \tag{3.2}$$

[2] This is a common modelling assumption. For example, see Epple et al. (2008), Fernández and Rogerson (2001) and Nechyba (2003). This assumption will be useful in Chapter 5.

Here, $\mu \geq 0$ is college quality. We could have had effort and other variables enter into this function as well, but we shall ignore those effects as they are not crucial for any of the following analysis. We now make the following assumptions about the college education process.

Assumption 3.3. *$k(\mu, g)$ is continuous and twice differentiable. Also, $k(0, g) = k(\mu, 0) = 0$, $\frac{\partial k(\mu,g)}{\partial \mu} \geq 0$, $\frac{\partial k(\mu,g)}{\partial g} \geq 1$ and $k(\mu, g) > g$ for $g, \mu \neq 0$.*

This last assumption implies that a college education will always increase human capital, and people with higher grades will experience a higher absolute increase in human capital.[3] This assumption should not be taken to mean that students with higher grades are 'better able' to take advantage of college education, as it is consistent with, say, a situation where college increases human capital by a fixed proportion.[4]

3.1.3 Individual Characteristics

To finish the description of the model, for each caste i, let a and p be distributed according to the joint distribution function $H_i(a, p)$. Let the marginal distribution of a be denoted $X_i(a)$ and let the conditional distribution of $p|a$ be denoted $B_i(p|a)$. We make the following crucial assumption which will be central to almost all of the following analysis.

Assumption 3.4. *(i) $H_i(\cdot, \cdot)$, $X_i(\cdot)$ and $B_i(\cdot)$ are continuous and have finite first moments; and (ii) $X_H(a) = X_L(a)$ and $B_H(p|a) \leq B_L(p|a)$ $\forall a, p$*

This states that ability is distributed evenly across castes, but for a given ability, H students are likely to have parents with more human capital than the L students' parents. In other words, the conditional distribution of p for H people first order stochastically dominates that for L people.[5] This captures the idea that the lower caste has been the victim of historical taste-based discrimination.

[3] Bertrand et al. (2008) report that going to engineering college increased the incomes of the low-caste college seat gainers (who have comparatively lower grades) by about Rs. 5000 less than it does for the displaced high-caste students (who have comparatively higher grades).

[4] A proportionate increase of between 40% and 70% is what is supported by the data in Bertrand et al. (2008).

[5] It is easy to see that this implies first order stochastic dominance of the marginal distribution as well.

3.2 Baseline Results without Affirmative Action

3.2.1 Distribution of Grades

Lemma 3.1. *Grades for caste i are distributed according to the distribution function $F_i(g) = E_a[B_i(P(a;g))]$ where $P(a;g)$ is implicitly defined by $g = g(q,a,P(a;g),e)$.*

Proof. Looking at the grades equation 3.1 and regarding q and e as fixed, the equation implicitly defines p as a function of a parameterized by g. Call this function $P(a;g)$. From Assumption 3.1, we know that the grades function is increasing in all its arguments. So, if one increased a, p would have to be reduced to achieve the same g. $P(a;g)$ is,thus, a decreasing function.

The probability that a grade will be below g is then the probability mass of $H_i(a,p)$ that lies to the left and below this function plotted in the $a-p$ space. This is calculated as follows.

$$F_i(g) = \int_a \int_0^{P(a;g)} h_i(a,p)\, dp\, da \tag{3.3}$$

Using the fact that any joint probability can be separated into a marginal and a conditional distribution, we can now write the following.

$$F_i(g) = \int_a \int_0^{P(a;g)} x_i(a) b_i(p|a)\, dp\, da \tag{3.4}$$

But the marginal does not depend on p, and so can be taken outside the second integral. Then, integrating gives us the following.

$$F_i(g) = \int_a x_i(a) B_i(P(a;g))\, da = E_a[B_i(P(a;g))] \tag{3.5}$$

□

Proving this result has equipped us to prove the following first order stochastic dominance result.

Theorem 3.1. $F_H(g) \leq F_L(g)\ \forall g$

Proof. Given Assumption 3.4, $B_H(P(a;g)) \leq B_L(P(a;g))$ for any $P(a;g)$. However, also due to Assumption 3.4, the marginal distribution of a is the same across castes. So $E_a[B_H(P(a;g))] \leq E_a[B_L(P(a;g))]$, which proves the result. □

3.2.2 The Cut-Off Grade

Having described the behavior of grades, we move on to describing what happens during college admissions. The 'cut-off' admissions rule can be formally described as follows. Take a cut-off grade g^*. Since a fraction $1 - F_H(g^*)$ of H people get grades above g^* and the total number of H is λn, the number of H admitted into college must be $(1 - F_H(g^*))\,\lambda n$. Similarly, we can get a number for L people. The total number of college admits must be $\bar{A}$. This gives us the following admissions equation which determines g^*.

$$[(1 - F_H(g^*))\,\lambda + (1 - F_L(g^*))\,(1 - \lambda)]\,n = \bar{A} \tag{3.6}$$

Theorem 3.2. *In the baseline model, without affirmative action, the H community is over-represented at the college level.*

Proof. From Theorem 3.1, $1 - F_H(g^*) \geq 1 - F_L(g^*)$, i.e. H students are more likely to get into college than L students. □

3.2.3 Realized Human Capital

For community i, average human capital given a cut-off level g^* is given by

$$\begin{aligned} \tilde{k}_i &= \int_0^{g^*} g\,dF_i(g) + \int_{g^*}^{\infty} k(\mu, g)\,dF_i(g) \\ &= \bar{g}_i + \int_{g^*}^{\infty} (k(\mu, g) - g)\,dF_i(g) \\ \text{where } \bar{g}_i &= \int_0^{\infty} g\,dF_i(g) \end{aligned} \tag{3.7}$$

This expression has the following intuitive interpretation. Expected human capital is the expected human capital produced by school in addition to the extra human capital obtained from a college education. Note that $\bar{g}_i$ is independent of g^*, i.e. if the cut-off grade is changed, only the additional returns from college will change; the returns from school remain the same.

Lemma 3.2. *The distribution of realized human capital is unequal, with $\tilde{k}_H \geq \tilde{k}_L$*

Proof. Integrating the second term in equation 3.7 by parts and applying Theorem 3.1 the result follows. □

Average human capital in the economy in this setting is given by

$$\begin{aligned}\tilde{k} &= \lambda \tilde{k}_H + (1-\lambda)\tilde{k}_L \\ &= \bar{g} + \left[\lambda \int_{g^*}^{\infty} (k(\mu,g)-g)\, dF_H(g) + (1-\lambda) \int_{g^*}^{\infty} (k(\mu,g)-g)\, dF_L(g)\right] \qquad (3.8)\\ &\text{where } \bar{g} = \lambda \bar{g}_H + (1-\lambda)\bar{g}_L\end{aligned}$$

This has a similar interpretation as above, with average human capital in the economy being human capital obtained from a school education plus the additional human capital obtained from a college education. Again, $\bar{g}$ is independent of g^*.

3.3 Affirmative Action

3.3.1 The Cut-Off Grades

In this model, as with all subsequent models, affirmative action is taken to mean a policy that achieves 'equal representation', i.e. the government mandates that the proportion of H and L people admitted to college must be the same. To achieve this, we would need two cut-off grades; g_H^* for H people and g_L^* for L people such that

$$1 - F_H(g_H^*) = 1 - F_L(g_L^*) \qquad (3.9)$$

In addition, the admissions equation becomes

$$\left[(1 - F_H(g_H^*))\,\lambda + (1 - F_L(g_L^*))\,(1-\lambda)\right] n = \bar{A} \qquad (3.10)$$

Together, equations 3.9 and 3.10 determine g_H^* and g_L^*.

Lemma 3.3. *As a result of affirmative action, the cut-off for H rises while the cut-off for L falls.*[6]

Proof. Without affirmative action, from Theorem 3.2 we have $1-F_H(g^*) \geq 1-F_L(g^*)$. Thus, in order to move from admissions equation 3.6 to admissions equation 3.10 we would need $1 - F_H(g^*) \geq 1 - F_H(g_H^*)$ and $1 - F_L(g^*) \leq 1 - F_L(g_L^*)$. However, given Theorem 3.1, this is only possible if $g_L^* \leq g^* \leq g_H^*$. □

[6]Bertrand et al. (2008) report that the cut-off scores for admission into engineering colleges covered in the study were 480/800 for the upper castes and 182/800 for the SC.

3.3.2 Realized Human Capital

Let us now move to the effect of affirmative action on realized human capital for each caste and its distribution between castes. New average human capital for caste i is given by

$$\tilde{k}_i^{AA} = \bar{g}_i + \int_{g_i^*}^{\infty} (k(\mu, g) - g)\, dF_i(g) \tag{3.11}$$

Theorem 3.3. *In the baseline model, affirmative action reduces inequality in human capital, but does not eliminate it.*

Proof. Average (and hence total) human capital for H falls since

$$\tilde{k}_H^{AA} - \tilde{k}_H = -\int_{g^*}^{g_H^*} (k(\mu, g) - g)\, dF_H(g) \leq 0 \tag{3.12}$$

Similarly, average (and hence total) human capital for L rises since

$$\tilde{k}_L^{AA} - \tilde{k}_L = \int_{g_L^*}^{g^*} (k(\mu, g) - g)\, dF_L(g) \geq 0 \tag{3.13}$$

However, the distribution of realized human capital is still unequal. To see this,

$$\tilde{k}_H^{AA} - \tilde{k}_L^{AA} = [\bar{g}_H - \bar{g}_L] + \left[\int_{g_H^*}^{\infty} (k(\mu, g) - g)\, dF_H(g) - \int_{g_L^*}^{\infty} (k(\mu, g) - g)\, dF_L(g)\right] \tag{3.14}$$

$[\bar{g}_H - \bar{g}_L] = M$ is positive given the first order stochastic dominance result we proved in Theorem 3.1. This is intuitive, as H students will get higher returns from school than L students. We must show that $\int_{g_H^*}^{\infty} (k(\mu, g) - g)\, dF_H(g) - \int_{g_L^*}^{\infty} (k(\mu, g) - g)\, dF_L(g) = N > 0$, i.e. the extra gains from college are also higher for H students.[7] To do this we integrate N by parts and use the fact that $1 - F_H(g_H^*) = 1 - F_L(g_L^*)$ to get

$$\begin{aligned} N = & [1 - F_L(g_L^*)][(k_H^* - g_H^*) - (k_L^* - g_L^*)] + \left[\int_{g_H^*}^{\infty} (k_g - 1)(1 - F_H(g))\, dg \right. \\ & \left. - \int_{g_H^*}^{\infty} (k_g - 1)(1 - F_L(g))\, dg \right] - \int_{g_L^*}^{g_H^*} (k_g - 1)(1 - F_L(g))\, dg \end{aligned} \tag{3.15}$$

Here, we have used $k_i^* = k(\mu, g_i^*)$ and $k_g = \frac{\partial k(\mu, g)}{\partial g}$ to make the representation more concise. Next, we observe that $\int_{g_H^*}^{\infty} (k_g - 1)(1 - F_H(g))\, dg - \int_{g_H^*}^{\infty} (k_g - 1)(1 - F_L(g))\, dg$ is positive as

[7] This is not as straightforward as before, as the integrals are over different ranges, and one might think that the extra range for L people might compensate for the stochastic dominance effect.

a straightforward application of Theorem 3.1. Also, as $1 - F_L(g)$ is weakly decreasing, and $k_g - 1 > 0$ from Assumption 3.3, we have

$$\begin{aligned}\int_{g_L^*}^{g_H^*} (k_g - 1)(1 - F_L(g))\,dg &\leq (1 - F_L(g_L^*)) \int_{g_L^*}^{g_H^*} (k_g - 1)\,dg \\ &= (1 - F_L(g_L^*))[(k_H^* - g_H^*) - (k_L^* - g_L^*)]\end{aligned} \tag{3.16}$$

Plugging this into equation 3.15, we get

$$N \geq \left[\int_{g_H^*}^{\infty} (k_g - 1)(1 - F_H(g))\,dg - \int_{g_H^*}^{\infty} (k_g - 1)(1 - F_L(g))\,dg\right] > 0 \tag{3.17}$$

We have thus shown that average human capital rises for L people and falls for H people, but the H people still enjoy a higher level of human capital after affirmative action. This is due to the fact that gains from a college education are higher in absolute terms for people with higher grades, and even with affirmative action, the H people that go to college have higher grades than the L people that go to college. □

Average human capital across the economy is now given by

$$\tilde{k}^{AA} = \bar{g} + \left[\lambda \int_{g_H^*}^{\infty} (k(\mu, g) - g)\,dF_H(g) + (1 - \lambda) \int_{g_L^*}^{\infty} (k(\mu, g) - g)\,dF_L(g)\right] \tag{3.18}$$

Theorem 3.4. *In the baseline model, affirmative action leads to a fall in economy-wide average human capital.*[8]

Proof. The introduction of affirmative action leads to college seats being taken away from H people with grades in the range $[g^*, g_H^*]$ and given to L people with grades in the range $[g_L^*, g^*]$. As the number of college seats is constant, we know that the number of people losing seats must be the same as the number of people gaining them. Mathematically,

$$\begin{aligned}\lambda n \int_{g^*}^{g_H^*} dF_H(g) &= (1 - \lambda) n \int_{g_L^*}^{g^*} dF_L(g) \\ \Leftrightarrow (k(\mu, g^*) - g^*) \int_{g^*}^{g_H^*} \lambda\,dF_H(g) &= (k(\mu, g^*) - g^*) \int_{g_L^*}^{g^*} (1 - \lambda)\,dF_L(g) \\ \Leftrightarrow \int_{g^*}^{g_H^*} \lambda(k(\mu, g) - g)\,dF_H(g) &> \int_{g_L^*}^{g^*} (1 - \lambda)(k(\mu, g) - g)\,dF_L(g)\end{aligned} \tag{3.19}$$

[8] This is supported by the result in Bertrand et al. (2008) that affirmative action policies come at an absolute cost.

We move from the second equality to the inequality because we know from Assumption 3.3 that $(k(\mu, g) - g)$ is increasing in g. So it must be true that $(k(\mu, g^*) - g^*) \int_{g^*}^{g_H^*} \lambda \, dF_H(g) < \int_{g^*}^{g_H^*} \lambda(k(\mu, g) - g) \, dF_H(g)$ and $(k(\mu, g^*) - g^*) \int_{g_L^*}^{g^*} (1 - \lambda) \, dF_L(g) > \int_{g_L^*}^{g^*} (1 - \lambda)(k(\mu, g) - g) \, dF_L(g)$

We can now apply inequality 3.19 as follows. The change in economy-wide average realized human capital is

$$\tilde{k}^{AA} - \tilde{k} = \int_{g_L^*}^{g^*} (1 - \lambda)(k(\mu, g) - g) \, dF_L(g) - \int_{g^*}^{g_H^*} \lambda(k(\mu, g) - g) \, dF_H(g) < 0 \tag{3.20}$$

$\square$

3.4 Summary of Main Results

In this chapter, we built a basic model of human capital formation through an education system with competition for a fixed number of college seats. To model the idea of the high-caste 'having it easier', we argued that parents' human capital is an input into their children's human capital formation at school, and that the distribution of the high-caste's human capital first order stochastically dominated that of the low-caste. This led to the distribution of grades for the high-caste children first order stochastically dominating that of the lower caste. Given the mechanical cut-off college admissions rule, this led to over-representation of high-caste children at college, and consequently, an inequality in human capital formation. This inequality thus stemmed from the unequal distribution of parents' human capital, and was magnified by the over-representation of the high-caste at college.

Affirmative action led to a redistribution of college seats. Some seats were transferred from the high to low-castes to achieve equal representation by choosing different cut-off grades for each caste. However, we found that while this reduced inequality, it did not eliminate it as even though the same proportion of high and low-caste students went to college, the high-caste students generally performed better because their parents generally had higher human capital. Also, we found that aggregate human capital reduced, as college seats were taken from high-caste students who were more 'able' to take advantage of a college education low-caste students who were less 'able'.

Chapter 4

Differences in School Quality

Many proponents of affirmative action argue that affirmative action is required to redress the imbalance of opportunities made available to children of different castes in their lives prior to college education. The starkest example of this is that high-caste children tend to go to 'good' schools whereas their low-caste counterparts go to 'bad' schools. In this section we allow schools to choose quality in order to loosely reproduce this stylized fact. Then, we look into the effects of affirmative action on school quality in addition to our previous outcome measures.

4.1 Model Set Up

4.1.1 School Education, College Education and Individual Characteristics

We retain the same definitions of $g(q, a, p, e)$, $k(\mu, g)$ and $H(a, p)$ used in Chapter 3, obeying Assumptions 3.1, 3.3 and 3.4. However, now q is allowed to vary, so we have the following simplifying assumption.

Assumption 4.1. *e is non-stochastic and constant across the population.*

4.1.2 Schools as Agents

For simplicity, let there be n schools, which will each educate one student.[1] Each school is free to choose its quality q in as much as it hires teachers and invests in physical infrastructure.

[1] The exact number of schools is not critical to the analysis. The analysis would proceed just as well with an arbitrary number of schools; it would only be slightly more cumbersome.

Assumption 4.2. *Costs of increasing quality are homogeneous*[2] *across schools at* $C(q)$ *with* $C(0) = 0$, $C'(q) > 0$ *and* $C''(q) > 0$.

This model will be driven by the incentives facing schools and their resultant choices of quality.

The payoff to a school is specified as follows. Schools look to maximize their reputation, which is measured by how well its alumni go on to perform in life. This can be motivated by the following logic. Schools with good reputations can usually charge higher fees. Moreover, they will attract more intelligent students and as their alumni do extremely well later in life, they can expect more donations or even state funding.[3]

In keeping with the reputation objective, we specify a school's payoff as the realized human capital of its student. However, as we shall specify later, when the school chooses its quality, it is only able to see the caste of applicants, not ability or parental income. Thus, we define the (prior) expected payoff of a school as

$$S(q; i, g_i^*) = E[\text{human capital}|i, g_i^*] - C(q) \tag{4.1}$$

The payoff obviously depends on the caste of the applicant because each caste has different distributions of grades. It also depends on the cut-off g_i^* as the cut off-determines whether a student goes to college or not, which affects human capital.[4] This payoff specification is a fairly important constituent of this model, as it means that schools will change their behaviour when affirmative action changes the likelihoods of students entering college, and therefore their attractiveness to the school.

4.1.3 Sequence of Actions

The sequence of actions is as follows.

1. n people are born, with nature assigning each person a caste, H with probability λ and L with probability $1 - \lambda$.

[2]It would be very easy to get heterogeneity in school quality via heterogenous costs. But here we want to abstract from that kind of heterogeneity and focus on the heterogeneity arising from the incentives faced by schools when they have to choose the student they must admit.

[3]Monks and Ehrenberg (1999) find that a college with a less favourable reputation (i.e. a low rank in the *U.S. News & World Report* College Rankings) leads to the entering class being of lower quality in terms of average SAT scores.

[4]Here we let the cut off grade depend on i as well, so that we do not have to rewrite this later when affirmative action kicks in. Without affirmative action, we have $g_H^* = g_L^* = g^*$.

2. One school (schools being indexed by i) is chosen at random and it chooses its quality q_i which is observed by all. This is repeated until all schools have chosen their qualities.[5]

3. Parents costlessly apply to all schools.[6]

4. Schools observe only caste[7] (not a or p) and decide on a preferred ranking. This ranking is used to make offers. The offer is first made to the candidate listed on the top, and if he rejects, then the offer is made to the next candidate. This process repeats itself till the seat is filled.

5. Parents choose which offer to take up.

6. Children undergo education at school and attain grades.

7. Colleges admit students based on the mechanical cut-off rule.

8. Those admitted undergo college education.

9. Payoffs are received.

Parents act on behalf of their children, and look to maximize k. This can be motivated by love for one's child, or more materialistic concerns such as having reliable support in old age.

4.2 Results without Affirmative Action

The model described above is essentially an extensive form game with $n + 2$ decision making nodes. Only steps 2 (a composite of n steps), 4 and 5 in the sequence of actions described above are decision making; the rest are mechanical. As with any extensive form game, we use the Subgame Perfect Equilibrium concept and solve via backwards induction. But before

[5]One would think that the more intuitive way of modelling school quality choice would be to have schools choose quality simultaneously. However, we have chosen sequential choice for two reasons. Firstly, as this is a one shot game, simultaneous choice does not allow schools to react to changes in other schools' qualities, thus artificially eliminating competition. Sequential choice allows competition in a one shot game. Secondly, it is closer to reality, where newer schools can observe the quality levels chosen by older schools.

[6]A variant of this model would have small applications costs, which would prevent parents from applying everywhere. If applications costs are small enough, we would get H parents applying to the λn schools with highest quality and L parents applying to the bottom $(1 - \lambda)n$ schools in terms of quality. However, even though this better simulates reality, it is an unnecessary complication that has no bearing on our results of interest.

[7]In societies such as India, a person's name is usually enough to give away his caste. So, this is not an unreasonable assumption. This is also not a bad assumption if we divided our population according to race instead of caste, as we would have to do in the case of countries like South Africa or several developed western countries.

doing so, we must derive results on the distribution of grades in a setting where q is allowed to vary.

4.2.1 Distribution of Grades

We first have the following result which is analogous to Lemma 3.1.

Lemma 4.1. *Grades for caste i are distributed according to the distribution function $F_i(g;q) = E_a\left[B_i\left(P(a;g,q)\right)\right]$ where $P(a;g,q)$ is implicitly defined by $g = g(q,a,P(a;g,q),e)$.*

Proof. Analogous to that of Lemma 3.1 except that since q varies, p as a function of a is also parameterized by q in addition to g. □

We now move on to first order stochastic dominance results on the distribution of grades.

Theorem 4.1. *(i) $F_H(g;q) \leq F_L(g;q)$ $\forall g$; (ii) $F_i(g;q'') \leq F_i(g;q')$ when $q'' \geq q'$*

Proof. For part (i), the proof is analogous to that of Theorem 3.1.

To prove part (ii), we proceed as follows. From Assumption 3.1 we know that g is increasing in all its arguments. For any given level of g, if we increase q, we would need a decrease in p to achieve the same g. Thus, we have $q'' \geq q' \Rightarrow P(a;g,q'') \leq P(a;g,q')$. But given the expression for $F_i(g;q)$ we derived in Lemma 4.1, this means that $F_i(g;q'') \leq F_i(g;q')$. As this holds for any level of g, this proves the result. □

4.2.2 Best Response Strategies

We now move on to characterizing the best response strategies in steps 2, 4 and 5 of the game through backward induction. For the purposes of the following derivations, let school j have chosen quality q_j.

Parents' Acceptance of Offers

Lemma 4.2. *Parents strictly prefer to accept an offer to not sending their child to school, and they prefer offers from high quality schools to offers from low quality schools. They are indifferent between offers from schools of same quality.*

Proof. Recall that parents wish to maximize their child's eventual human capital. To prove the first part, we must assume (quite reasonably) that if a student does not go to school, he cannot go to college. In such a setting, from Assumption 3.1 the child's human capital is

$g(0, a, p, e) = 0$. Since g is increasing in q, a parent will prefer to send his child to school, as even if the child ends up not getting into college, his eventual human capital will be at least as large as 0, given that quality is weakly positive.

To prove the second part, note that eventual human capital (given a g^*) is given by

$$\theta(q) = \begin{cases} g(q, a, p, e) & \text{if } g < g^* \\ k(\mu, g(q, a, p, e)) & \text{if } g > g^* \end{cases} \tag{4.2}$$

Here, remember that the parent knows (a, p) for his child and e is constant. We know from Assumptions 3.1 and 3.3 that g is increasing in q and that k is increasing in g. So, θ is increasing in q. In other words, if school quality increases, the child's return from a school and a college education increases, and there is also an increased possibility that the child will be able to go to college. So, a parent will prefer to accept an offer from a higher quality school over an offer from a lower quality school.

It is also apparent that the payoff to the parent from accepting offers from schools with the same quality is the same, hence he will be indifferent between such offers. □

Schools' Offers

Lemma 4.3. *Schools will rank high-caste students above low-caste students, but within castes the school is indifferent between students and will therefore rank them randomly.*

Proof. We begin by noting that the school receives no information about applicants apart from caste. So, the decision on ranking preference must be based only on caste. School j's expected payoff from enrolling an applicant of caste i can be rewritten as follows.

$$\begin{aligned} S(q_j; i, g_i^*) &= P(\text{student gets into college}|i) \cdot E[\text{human capital}|i, g_i^*, \text{gets into college}] + \\ &\quad P(\text{doesn't get into college}|i) \cdot E[\text{human capital}|i, g_i^*, \text{doesn't get into} \\ &\quad \text{college}] - C(q_j) \\ &= (1 - F_i(g_i^*; q_j)) \frac{\int_{g_i^*}^{\infty} k(\mu, g)\, dF_i(g; q_j)}{1 - F_i(g_i^*; q_j)} + F_i(g_i^*; q_j) \frac{\int_0^{g_i^*} g\, dF_i(g; q_j)}{F_i(g_i^*; q_j)} - C(q_j) \\ &= \bar{g}_i + \int_{g_i^*}^{\infty} (k(\mu, g) - g)\, dF_i(g; q_j) - C(q_j) \end{aligned} \tag{4.3}$$

Note that the first two terms together are exactly the same as $\tilde{k}_i$ (in equation 3.7), i.e. the expected human capital for caste i. Just as before, it is now a simple exercise to integrate by parts and apply the stochastic dominance result (Theorem 4.1) to see that $S(q_j; H, g^*) \geq S(q_j; L, g^*)$.

Thus, a school would prefer to rank high-caste people above low-caste people, but within a caste, expected payoff is the same, so schools are indifferent between applicants of the same caste. Of course, given that the schools have chosen quality already, they can work out through backward induction whether their first choice candidate will accept the offer or not, but since making offers doesn't cost anything, this should not deter schools from stating their true preferred ranking. $\square$

Schools' Quality Choice

Naturally, the schools look to maximize their expected payoff. In order to avoid uninteresting equilibria like all schools choosing a quality of 0, or all schools indefinitely raising quality, we must make the following regularity assumptions.

Assumption 4.3. *(i) After some quality level, costs outweigh and increase faster than benefits; (ii) for at least some range of quality $S(q_j; i, g_i^*) > 0$.*

Part (i) ensures that $\lim_{q_j \to \infty} S(q_j; i, g_i^*) = -\infty$ and part (ii) ensures that there will be a positive quality level where the school would want to function (i.e. choose a quality level greater than 0).

Let us now find out what the expected payoff curves for high and low-caste students look like, graphically. It is impossible to state the exact shape without specifying functional forms for the distributions and human capital production functions. However, we can make the following observations.

- From Assumption 3.1, $S(0; i, g_i^*) = 0$.
- From Assumption 4.3, there is some range over which $S(q_j; i, g_i^*) > 0$ and eventually $S(q_j; i, g_i^*)$ falls below 0 and tends to $-\infty$ in the limit.
- From the proof of Lemma 4.3, $S(q_j; H, g^*) \geq S(q_j; L, g^*)$. This implies that expected payoff curve given a high-caste student lies above the expected payoff curve given a low-caste student for every quality level.

Keeping these broad parameters in mind, the expected payoff curves given high and low-caste students can be drawn as in Figure 4.1. Here, we have drawn the curves so that the range

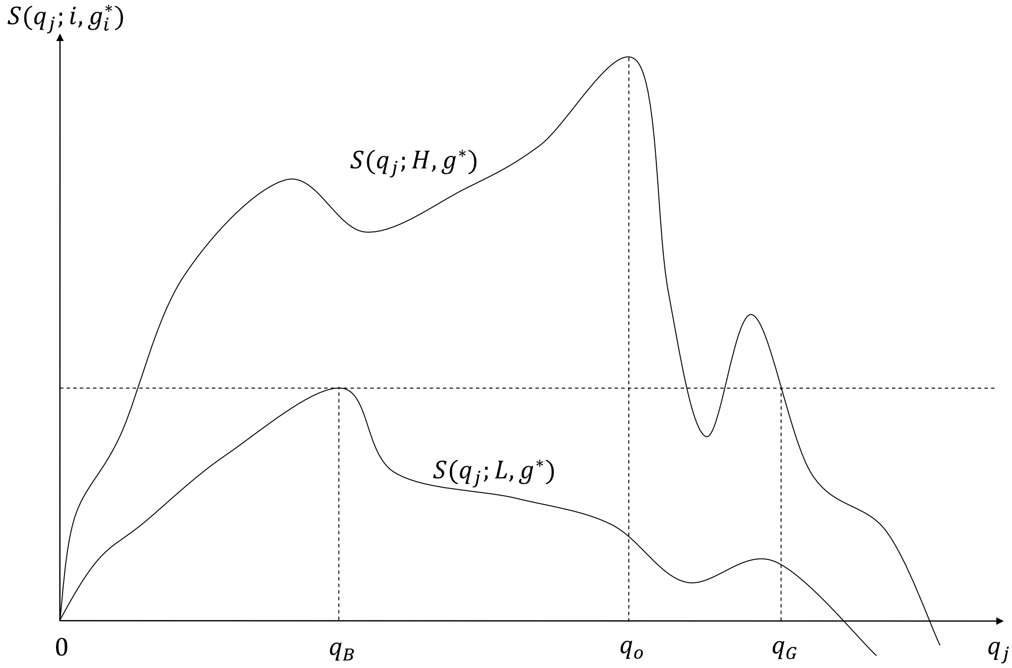


Figure 4.1: Schools' Quality Choice

for which $S(q_j; i, g_i^*) > 0$ occurs directly after 0; it need not be so. We must first note that these curves are not 'well behaved', in the sense that they are not necessarily single peaked or concave. It is important to understand why this is so, and why no simple assumption can be invoked to rule this out. The expected payoff curve for caste i has the following slope.

$$\begin{aligned}\frac{\partial S(q_j; i, g_i^*)}{\partial q_j} = -\Bigg[&\int_0^{g_i^*} g\frac{\partial F_i(g_i^*; q_j)}{\partial q_j}\, dg + (k(\mu, g_i^*) - g_i^*)\frac{\partial F_i(g; q_j)}{\partial q_j} \\ &+ \int_{g_i^*}^{\infty} k(\mu, g)\frac{\partial F_i(g; q_j)}{\partial q_j}\, dg + C'(q_j)\Bigg]\end{aligned} \tag{4.4}$$

Also, from the proof of Lemma 4.1, we have

$$F_i(g; q) = \int_a B_i(P(a; g, q))\, da \Leftrightarrow \frac{\partial F_i(g; q)}{\partial q} = \int_a b_i(P(a; g, q))\frac{\partial P(a; g, q)}{\partial q}\, da \tag{4.5}$$

We know from the proof of Theorem 4.1 that $\frac{\partial P(a;g,q)}{\partial q} \leq 0$.[8] Consider now a simple bell shaped density function for $b_i(\cdot)$. A low value of q would correspond to a high value of $P(a; g, q)$, which, in a bell shaped density would mean a small value of $b_i(P(a; g, q))$, and so a small negative value of $\frac{\partial F_i(g;q)}{\partial q}$. However, it is entirely reasonable that $C'(q)$ is low enough here for the slope to be positive. It is also reasonable that as we increase q, $C'(q)$ increases

[8] Since we know that $b_i(P(a; g, q)) \geq 0$, this confirms our result in Theorem 4.1 that $F_i(g; q)$ is decreasing in q.

but the reduction in $P(a; g, q)$ does not translate into a massive increase in $b_i(P(a; g, q))$ (as we could still be in the right tail), and thus $\frac{\partial F_i(g;q)}{\partial q}$ does not turn significantly more negative. Thus the slope turns negative. Now, when we increase q enough for us to move into the middle of the distribution, then we have a large $b_i(P(a; g, q))$ leading to a large negative $\frac{\partial F_i(g;q)}{\partial q}$. It could be large enough to overtake $C'(q)$, and the slope could turn positive again. This would undoubtedly lead to at least two peaks.

However, our results are invariant to this 'misbehaviour'; they would be exactly the same with single-peaked concave expected payoffs.

Next, we must note that these curves are in actuality upper and lower bounds of a family of curves, which represent the schools' expected utility when it may admit a high or low-caste applicant with some probability combination. The higher the probability of admitting a high-caste student, the closer would the curve be to the upper bound. As is depicted in Figure 4.1, call the quality levels where $S(q_j; L, g^*)$ and $S(q_j; H, g^*)$ are maximized q_B and q_0 respectively. Also, call the quality level where $S(q_j; H, g^*)$ goes below $\max S(q_j; L, g^*)$ for the last time q_G.

Lemma 4.4. *The schools' best response strategies are as follows.*
(i) For each of the last λn schools, if there already λn schools at or above quality q_G, it chooses q_B. Else, it chooses the quality $\arg\max_{q \in Q} S(q; H, g^*)$ where Q is the set of values of q that guarantee the school a place in the final top λn.
(ii) For each of the first $(1-\lambda)n$ schools, if there are already λn schools at or above quality q_G, then it chooses q_B, else it chooses q_G.

Proof. We first establish that, given Lemmas 4.2 and 4.3, a school must be in the top λn in terms of quality to ensure that it gets an H student. This can be seen as follows. All schools will place H students above L students in their rankings. So all schools will first make offers to H students. As H students have first choice, they will accept the highest quality offer made to them. It is then easy to see that the H students will be distributed among the top λn schools.

Now consider the decision of the last school to be drawn. The other $n-1$ schools have already chosen their qualities. The school knows that there are λn H students, and that if it chooses a quality that puts it in the top λn, it is guaranteed an H student. Now, if the lowest quality among the top λn till then is less than q_G, then it is possible, by choosing a higher value, to attract an H student and get a payoff that is greater than the maximum payoff from obtaining an L student. However, if the lowest quality among the previous top λn is at or beyond q_G, then by choosing a higher quality (and thereby attracting an H student) it gets a lower expected payoff than if it chooses q_B and settles for an L student.

Note the special case where the lowest of the previous λn is exactly q_G. It is clear that the school would not want to chose a quality greater than q_G, but it will also not want to choose a quality equal to q_G. This is because doing so would mean that there is no distinct top λn, and so it will receive an H student with some probability, which could place it on a lower curve and give it a payoff strictly lower than it would get if it chose q_B.

We now move to the $(n-1)$th school to be drawn. It knows that the last school will definitely choose a quality up to q_G in a bid to attract an H student. Thus, in order to ensure that it finishes in the final top λn and gets an H student, it must ensure that its quality choice places it in the top $(\lambda n - 1)$ out of $(n-1)$. If, however, the lowest of the previous top $(\lambda n - 1)$ is at q_G, then it should choose q_G as well, since it knows that the next school when faced with the situation will chose q_B. If, however, there are already λn schools at or above q_G, then it does best to choose q_B, by the earlier argument.

A similar pattern carries on with the $(n-2)$th school wanting to be in the top $(\lambda n - 2)$ and so on until the $(n - \lambda n)$th school. Consider the case when the lowest of the previous top λn is less than q_G. This school knows that there are λn schools to come after it, so if it chooses a quality less than q_G the subsequent schools will surely end up choosing qualities higher. The only way it can guarantee getting an H student is by choosing q_G, as it knows that then the subsequent $(\lambda n - 1)$ schools would all also choose q_G and the last one would choose q_B. If, however, there are already λn schools at q_G or beyond, then it would choose q_B as before. This reasoning holds for the rest of the chain right up to the first school. $\square$

4.2.3 Characterizing the Equilibrium

Theorem 4.2. *In equilibrium,*

(i) the first λn schools choose quality q_G.

(ii) the last $(1-\lambda)n$ schools choose quality q_B.

(iii) every school ranks H applicants above L applicants while making offers, but randomizes within applicants from the same caste.

(iv) all H students are randomly distributed among good quality schools and all L students among bad quality schools.

Proof. Given the best response strategies derived in Lemmas 4.2, 4.3 and 4.4, the result immediately follows.

To verify that this is an equilibrium, we can check that at no stage would an agent like to deviate. In step 2, consider any of the first λn schools. It is plain from looking at Figure 4.1 that raising quality would mean a lower payoff and lowering quality would mean getting a low-caste student for sure (thus placing it on the lower curve), as given the best response strategies the $(\lambda n + 1)$th school would choose quality q_G, thus excluding it from the top λn schools. The best deviation is to choose q_B, which gives it the same payoff as it gets by choosing q_L.

For any of the last $(1 - \lambda)n$ schools, being on the lower curve is certain given that the first λn have chosen q_H. Then, it is apparent that any deviation will bring a strictly lower payoff than choosing q_L.

In step 4, any of the first λn schools gets a strictly lower payoff from by offering a seat to an L candidate before an H candidate (as then with a certain probability it will get an L student) while any of the last $(1 - \lambda)n$ get the same payoff (as it cannot hope to ever attract an H student).

In step 5, it is clear that a parent would do worse by accepting an offer from a bad quality school over one from a good quality school. □

4.2.4 The Cut-Off Grade

The admissions rule is the same as in the baseline model, but the distribution of grades is slightly different (from Lemma 4.1). The probability that a student of caste i going to school with quality q_j attains the cut-off grade g^* is $F_i(g^*; q_j)$. Since we know that all H students go to good quality schools and all L students go to bad quality schools, the admissions equation may be written as

$$[(1 - F_H(g^*; q_G))\,\lambda + (1 - F_L(g^*; q_B))\,(1 - \lambda)]\,n = \bar{A} \tag{4.6}$$

Theorem 4.3. *In the differences in school quality model, without affirmative action, the H community is over-represented at the college level.*

Proof. From Theorem 4.1, $1 - F_H(g^*; q_G) \geq 1 - F_L(g^*; q_G) \geq 1 - F_L(g^*; q_B)$. □

4.2.5 Realized Human Capital

Analogous to the baseline model, average human capital for community i given a cut-off grade g^* is

$$\begin{aligned} \tilde{k}_i &= \bar{g}_i + \int_{g^*}^{\infty} (k(\mu, g) - g)\ dF_i(g; q_j) \\ \text{where } \bar{g}_i &= \int_0^{\infty} g\, dF_i(g; q_j) \\ \text{and } j &= \begin{cases} G & \text{if } i = H \\ B & \text{if } i = L \end{cases} \end{aligned} \tag{4.7}$$

Lemma 4.5. *The distribution of realized human capital is unequal with $\tilde{k}_H \geq \tilde{k}_L$.*

Proof. This may be seen by integrating by parts and applying Theorem 4.1. □

Average human capital in the economy is now given by

$$\begin{aligned} \tilde{k} &= \bar{g} + \left[\lambda \int_{g^*}^{\infty} (k(\mu, g) - g)\, dF_H(g; q_G) + (1-\lambda) \int_{g^*}^{\infty} (k(\mu, g) - g)\, dF_L(g; q_B)\right] \\ &\text{where } \bar{g} = \lambda \bar{g}_H + (1-\lambda)\bar{g}_L \end{aligned} \tag{4.8}$$

This is analogous to the baseline model.

4.3 Affirmative Action

As earlier, affirmative action involves the choosing of two cut-off grades, g_H^* and g_L^* for the low and high-castes respectively, such that there is equal representation in college. Let us first examine the impact of this on the best response strategies, and then on realized outcomes.

4.3.1 Best Response Strategies

Parents' Acceptance of Offers

Lemma 4.6. *Affirmative action does not change parents' strategies in accepting offers. The strategies are the same as given in Lemma 4.2.*

Proof. The probability of getting into college does not enter the proof of Lemma 4.2. The only difference with respect to that proof is that g^* is replaced by g_i^*. □

Schools' Offers

We first see what happens to $S(q_j; i, g_i^*)$ as g_i^* changes.

Lemma 4.7. *$S(q_j; i, g_i^*)$ rises as g_i^* falls and vice versa.*

Proof. Differentiating equation 4.3 with respect to g_i^*, we get

$$\frac{\partial S(q_j; i, g_i^*)}{\partial g_i^*} = -\left[k(\mu, g_i^*) - g_i^*\right] f_i(g_i^*; q_j) < 0 \tag{4.9}$$

The last inequality follows from Assumption 3.3. □

So, if $g_H^* \geq g^* \geq g_L^*$, the expected payoff curve for the L people moves up at every point and that for the H people moves down at each point. If the schools choose the same quality, then we can prove that $S(q_j; H, g_H^*) \geq S(q_j; L, g_L^*)\ \forall q_j$. The proof is exactly the same as that for the last part of Theorem 3.3. However, in this model, schools chose two different quality levels without affirmative action. It is conceivable that, if the L people all end up going to bad quality schools, the cut-off grade for them would have to fall very low in order to achieve equal representation. Then it may be more profitable for a good quality school to take an L candidate in, given that, with the high quality schooling and the low cut off grade, the candidate is almost sure to get into college, and may end up having higher expected human capital than an H candidate for whom the cut-off is really high.

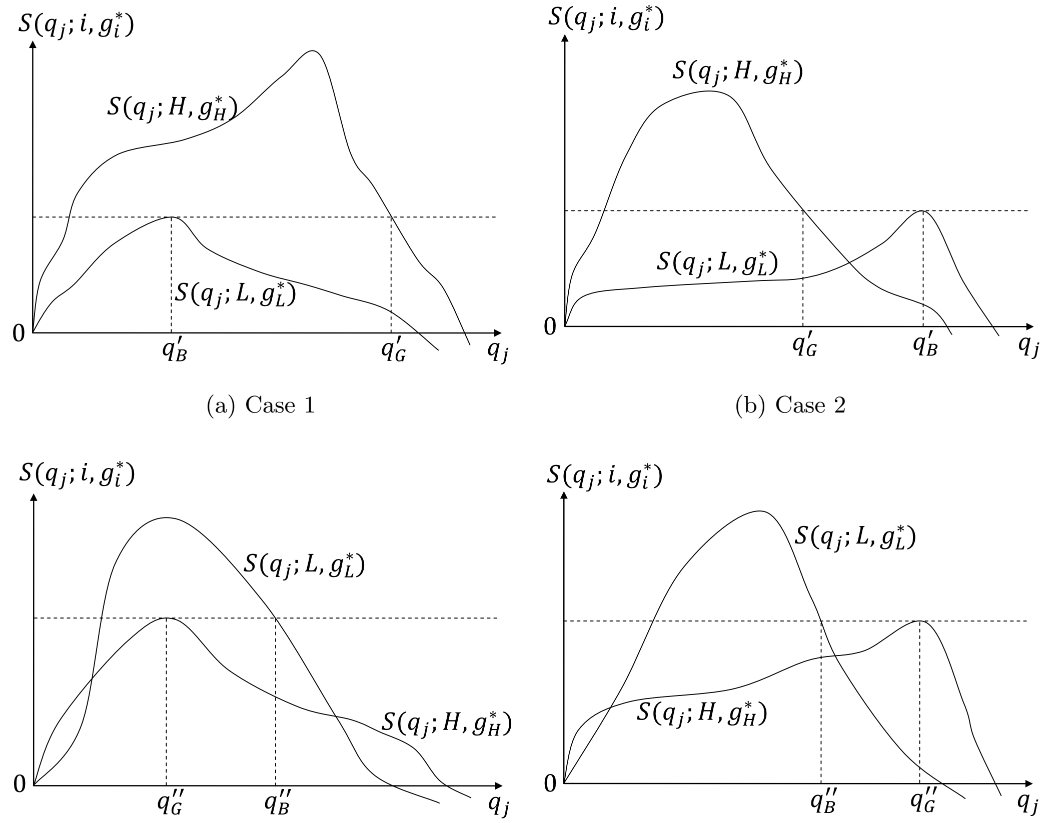
Thus, we could have situations when certain schools would prefer to rank L students above H students. We know from previous analysis that the equilibrium is determined by the lower maximum and the rightmost point on the other curve that corresponds to the same level of expected payoff. If the lower maximum is that of the L curve, call the quality level where $S(q_j; L, g_L^*)$ is maximized q'_B, and call the quality level where $S(q_j; H, g_H^*)$ goes below $\max S(q_j; L, g_L^*)$ for the last time q'_G. If, however, the lower maximum is that of the H curve, call the quality level where $S(q_j; H, g_H^*)$ is maximized q''_G, and call the quality level where $S(q_j; L, g_L^*)$ goes below $\max S(q_j; H, g_H^*)$ for the last time q''_B. With these definitions, we can have the following four configurations, which are depicted in Figure 4.2.

1. $\max S(q_j; H, g_H^*) \geq \max S(q_j; L, g_L^*)$ and $q'_G \geq q'_B$.

2. $\max S(q_j; H, g_H^*) \geq \max S(q_j; L, g_L^*)$ and $q'_G \leq q'_B$.

3. $\max S(q_j; H, g_H^*) \leq \max S(q_j; L, g_L^*)$ and $q''_G \leq q''_B$.

4. $\max S(q_j; H, g_H^*) \leq \max S(q_j; L, g_L^*)$ and $q_G'' \geq q_B''$.



(a) Case 1

(b) Case 2

(c) Case 3

(d) Case 4

Figure 4.2: Possible Configurations of Expected Payoff Curves After Affirmative Action

We will only discuss the first in detail, as the rest have a similar mode of analysis and the first case gives the most intuitive and realistic result.

In the first case, we know that at least at quality q_G' schools will rank H students over L students. A more detailed depiction of Case 1 in relation the the situation without affirmative action is provided in Figure 4.3. The way we have drawn it, the schools with quality q_B' also prefer to rank H students over L students, but we can conceive of examples where this ranking could be reversed, though it will make no difference to the outcome.

Schools' Quality Choice

Lemma 4.8. *The schools' best response strategies do not change due to affirmative action. The strategies are as given in Lemma 4.4, with q_B and q_G being replaced by q_B' and q_G' respectively.*

Proof. Given that $\max S(q_j; H, g_H^*) \geq \max S(q_j; L, g_L^*)$ and $q_G' \geq q_B'$, we know that at least

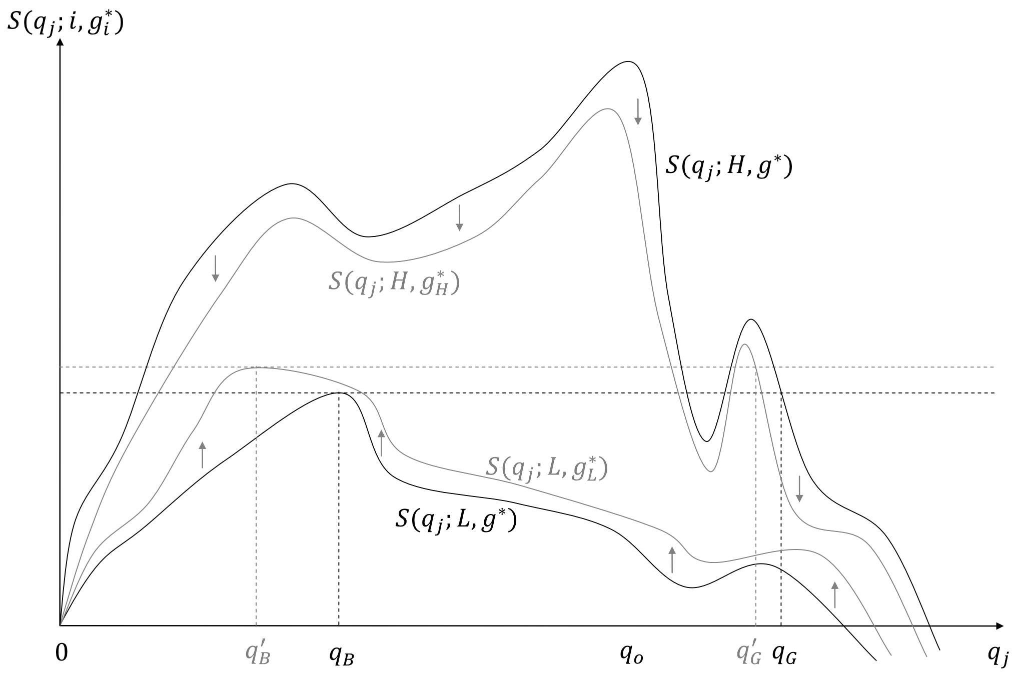


Figure 4.3: Schools' Quality Choice Under Affirmative Action

over a range of q_j, H students give a higher payoff. We also know that in order to attract H students a school must be in the top λn and must rank H students over L students. The proof is therefore similar in all respects to that of Lemma 4.4, because we know at q_G' schools will rank H students above L students. □

4.3.2 Characterizing the Equilibrium

Theorem 4.4. *In equilibrium,*

(i) the first λn schools choose quality q_G'.

(ii) the last $(1-\lambda)n$ schools choose quality q_B'.

(iii)(a) the top λn schools rank H applicants above L applicants while making offers, but randomize within applicants from the same caste.

(iii)(b) if (as in Figure 4.3) $S(q_B'; H, g_H^*) \geq S(q_B'; L, g_L^*)$, the last $(1-\lambda)n$ schools rank H applicants above L applicants; if $S(q_B'; H, g_H^*) \leq S(q_B'; L, g_L^*)$, the last $(1-\lambda)n$ schools rank L applicants above H applicants. Whatever the ranking between castes, the within caste ranking is random.

(iv) all H students are randomly distributed among good quality schools and all L students among bad quality schools.

Proof. Given that $\max S(q_j; H, g_H^*) \geq \max S(q_j; L, g_L^*)$ and $q_G' \geq q_B'$, the result follows from

Lemmas 4.6 and 4.8. We can also verify the equilibrium by checking that at no stage would any stage like to deviate, which can be done exactly as in the proof of Theorem 4.2 □

4.3.3 The Cut-Off Grades

Given the equilibrium characterization, the equal representation condition becomes

$$1 - F_H\left(g_H^*; q_G'\right) = 1 - F_L\left(g_L^*; q_B'\right) \tag{4.10}$$

The admissions constraint becomes

$$\left[\left(1 - F_H\left(g_H^*; q_G'\right)\right)\lambda + \left(1 - F_L\left(g_L^*; q_B'\right)\right)(1-\lambda)\right] n = \bar{A} \tag{4.11}$$

Together, equations 4.10 and 4.11 determine g_H^* and g_L^*.

Lemma 4.9. *If the new school qualities are 'close enough' to the old ones, affirmative action leads to a rise in the cut-off for H students and a fall for L students.*

Proof. Without affirmative action, from Theorem 4.3, we have $1-F_H(g^*; q_G) \geq 1-F_L(g^*; q_B)$. In order to move from such a situation to equal representation, the probability of a H student getting in must fall and that of an L student getting must rise. Se we would need $1 - F_H(g_H^*; q_G') \leq 1 - F_H(g^*; q_G)$ and $1 - F_L(g_L^*; q_B') \geq 1 - F_L(g^*; q_B)$. From Theorem 4.1, we know that $1 - F_H(g^*; q_G) \geq 1 - F_H(g_H^*; q_G')$, and so if q_G' is not 'too far' below q_G, then $1 - F_H(g_H^*; q_G') \leq 1 - F_H(g^*; q_G)$ would imply $g_H^* \geq g^*$. Similarly, if q_B' is not 'too far' above q_B, then $g_L^* \leq g^*$. □

4.3.4 What Happens to School Quality?

Good Quality Schools

Theorem 4.5. *Good quality schools necessarily undergo a reduction in quality as a result of affirmative action.*

Proof. We know from Lemma 4.7 that if the cut-off for H students rises, the expected payoff curve moves down at every point. Also, if the cut-off for L students falls, the expected payoff curve moves up at every point. The quality choice of the good quality schools is determined by drawing a straight line through the maximum of the expected payoff curve given an L

student and seeing where it intersects the expected payoff curve given an H student for the last time. Since the lower curve moves up, so does its maximum and the straight line through it. The last intersection of this line with the original higher curve is then necessarily at a lower quality level than previously, as at the last intersection the higher curve is negatively sloped (it must reach $-\infty$ at extremely high quality levels). But the higher curve also moves down, so the last intersection with the new higher curve must be at an even lower quality level.

Intuitively, this must be interpreted as follows. Affirmative action makes L students more profitable, so the top λn schools are less willing to compete for H students. Also, H students become less profitable as it is less likely that they will get into college. □

A Monotone Comparative Statics Result

We now move to the effect on the quality of bad quality schools. Since the quality of these schools is determined by the maximum of the expected payoff curve for L applicants, we must determine how this maximum moves as a result of affirmative action. To do this, we appeal to the following version of a standard monotone comparative statics result, which can be found in Milgrom and Shannon (1994).

Lemma 4.10. *Let X be a lattice, let T be a partially ordered set and let the function $f : X \times T \rightarrow \mathbb{R}$ have the following properties:*
(i) $\forall t \in T$, $f(\cdot, t)$ is quasi-supermodular (QSM) in x.
(ii) $f(x,t)$ has single crossing differences (SCD) from negative to positive.
Then $\arg\max_{x\in X} f(x,t'') \geq \arg\max_{x\in X} f(x,t')$ if $t'' > t'$. The converse holds if the SCD is positive to negative.

Proof. See Milgrom and Shannon (1994). □

In order to be able to understand and consequently apply this result, we need to go through the following definitions of the key terms involved.

A set $(S, \geq)$ is a **partially ordered set** if the following hold $\forall x, y, z \in S$

1. $x \geq x$ (reflexivity)

2. $x \geq y$ & $y \geq x \Rightarrow x = y$ (anti-symmetry)

3. $x \geq y$ & $y \geq z \Rightarrow x \geq z$ (transitivity)

An element $a \in S$ is an **upper bound (lower bound)** of $S' \subseteq S$ if $a \geq (\leq) s' \; \forall s' \in S'$.

a^* is the **least upper bound / supremum (greatest lower bound / infimum)** of $S' \subseteq S$ when a^* is an upper bound (lower bound) of S' and for any other upper bound (lower bound) of S', say $\bar{a}$, $a^* > (<) \bar{a}$.

A partially ordered set $(S, \geq)$ is a **lattice** if every two element subset of S has an infimum and a supremum.

A function $f : X \to \mathbb{R}$ is **QSM** if $f(x) - f(x \wedge x') \geq 0 \Rightarrow f(x \vee x') - f(x') \geq 0$ where $f(x \wedge x')$ is the infimum and $f(x \vee x')$ is the supremum of x and x'.

A function $f : X \times T \to \mathbb{R}$ has **SCD** if whenever $x'' > x'$, the function $g(t) = f(x'', t) - f(x', t)$ has the single crossing property, i.e. it crosses 0 only once, from negative to positive. It is apparent that a sufficient condition for SCD to be satisfied is for f to have **increasing differences (ID)**, or for $g(t)$ to be increasing in t. It can also be shown that if $X \subseteq \mathbb{R}^l$ and $T \subseteq \mathbb{R}^k$ then $f(x, t)$ has ID if

$$\frac{\partial^2 f}{\partial x_i \partial t_j} \geq 0 \; \forall i = 1, \ldots, l, \; j = 1, \ldots, k \tag{4.12}$$

If this inequality is reversed, we get decreasing differences, which corresponds to SCD when $g(t)$ crosses 0 only once, but from positive to negative.

Here, x is the set of choice variables and t is the set of parameters. The result says that the value of X that globally maximizes f is increasing in the parameters t provided two technical conditions (QSM and SCD) are met. Note, the result does not rely on continuity or concavity.

Bad Quality Schools

We are now in a position to apply this result to find out how affirmative action affects the quality of the bad quality schools.

Theorem 4.6. *If* $\frac{\partial f_L(g; q_j)}{\partial q_j} \leq 0$ *in the vicinity of* g^**, quality will fall or stay constant as a result of affirmative action. If the opposite holds, quality will rise.*

Proof. We get this result by applying Lemma 4.10 to the expected payoff function given an L student. In order to do so, we first note that our choice variable is $q_j \in \mathbb{R}_+$, and we take our parameter set to be the cut-off grade. We treat the cut off grade as a variable $\hat{g} \in \mathbb{R}_+$, which takes the values g^* without affirmative action and g_L^* with affirmative action. Then,

$S(q_j; L, \hat{g}) : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ as required by Lemma 4.10. Moreover, we observe that $\mathbb{R}_+$ satisfies the properties of both a lattice and, defining '$\geq$' as the euclidian relation, a partially ordered set.

Now, we need to check the two conditions of QSM and SCD. However, any function where the set of choice variables is scalar is always QSM in that variable. This can be easily verified by checking that the conditions of QSM are satisfied.

Thus, we only need to check SCD, but we know that a sufficient property for that is ID, which implies

$$\begin{aligned} &\frac{\partial^2 S(q_j; L, \hat{g})}{\partial q_j \partial \hat{g}} \geq 0 \\ \Leftrightarrow -\left(k\left(\mu, \hat{g}\right) - \hat{g}\right) &\frac{\partial f_L\left(\hat{g}; q_j\right)}{\partial q_j} \geq 0 \\ \Leftrightarrow &\frac{\partial f_L\left(\hat{g}; q_j\right)}{\partial q_j} \leq 0 \end{aligned} \tag{4.13}$$

Here, we know that $k(\mu, g) - g > 0$ from Assumption 3.3. We also know that initially, without affirmative action, $\hat{g} = g^*$, so we must have $f_L\left(g; q_j\right)$ decreasing in q_j in the vicinity of g^* for SCD to hold. Now, a straightforward application of Lemma 4.10 tells us that if this condition holds, school quality will move in the same direction as the cut-off grade. □

The intuition behind this result can be gained with the help of Figure 4.4. Take for sim-

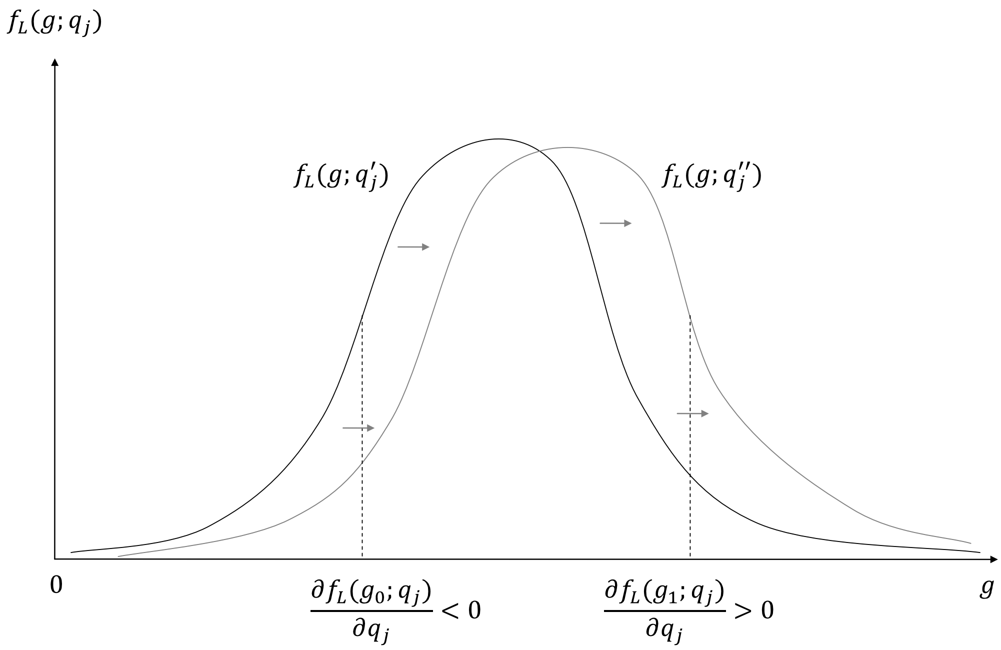


Figure 4.4: The Effect of $f_L(g, q_j)$ on School Quality

plicity a single peaked density function $f_L(g; q_j)$ at some quality q_j'. If we let quality increase to q_j'', we know from Theorem 4.1 that the new distribution will stochastically dominate the old one. So the curve would move in a manner similar to the way shown in the diagram. There will be some grades that will experience an increase of probability mass (like g_1), whereas some will experience a decrease (like g_0).

Now, if the cut-off grade is at g_1, then an increase in quality will increase the probability mass around the cut-off grade. If this happens, then the reduction in cut-off caused by affirmative action will increase the probability of the student getting into college much faster. However, if the cut-off is at a point where an increase in quality reduces probability mass (like g_0), then an increase in quality reduces the effectiveness of a falling cut-off in increasing the probability of getting into college. The incentive here is to reduce quality to again put more mass in the vicinity of the cut-off, so as to take maximum advantage of the fact that the cut-off is decreasing. Thus, schools adjust quality to put more people in the vicinity of the cut-off, so as to better exploit the effect that a falling cut-off has in increasing the prospects of the students getting in to college.

4.3.5 Realized Human Capital

Theorem 4.7. *The distribution of human capital is still unequal despite affirmative action. Moreover, it is ambiguous whether inequality in human capital is reduced; under certain conditions it may increase.*

Proof. Remember we are still dealing with the case where $\max S(q_j; H, g_H^*) \geq \max S(q_j; L, g_L^*)$ and $q_G' \geq q_B'$. If we look at the extreme where $q_G' = q_B'$, then we can show in a manner analogous to the proof of Theorem 3.3 that inequality still persists. Then we note that $\tilde{k}_L$ is decreasing in school quality (this can be easily verified through differentiating and using the first order stochastic dominance result, Theorem 4.1), and so if the quality of bad schools is less than q_G' the average realized human capital will be less, which would increase the inequality. This proves the first statement.

To prove the second statement, we note that, firstly, average human capital for H people unambiguously falls as school quality reduces and it is more difficult to get into college as the cut off has increased. Formally,

$$\begin{aligned}\tilde{k}_H^{AA} - \tilde{k}_H = &\int_0^\infty \left(F_H(g; q_G) - F_H(g; q_G')\right) dg + \left[(k(g_H^*, \mu) - g_H^*)(1 - F_H(g_H^*; q_G'))\right. \\ &- \left.(k(g^*, \mu) - g^*)(1 - F_H(g^*; q_G))\right] - \int_{g^*}^{g_H^*} (k(\mu, g) - g)\, dF_H(g; q_G) \\ &+ \int_{g_H^*}^\infty (k_g - 1)\left(F_H(g; q_G) - F_H(g; q_G')\right) dg \\ &\leq 0\end{aligned} \tag{4.14}$$

Here, we have used Theorem 4.1 and Assumption 3.3, and we know that $q_G' < q_G$. Secondly, we note that the effect on effect on average human capital for L people is ambiguous, because even though the probability of getting into school increases (which has the effect of increasing human capital), it is possible for school quality to decrease (which would lower human capital). Indeed, if school quality decreases enough, it may more than offset the increase due to the lowering of the cut-off grade, thus increasing inequality.

We can say though, that if bad school quality increases, then human capital for L will increase, leading to a reduction in inequality. This can be shown formally similarly to the high-caste case. □

Theorem 4.8. *The effect on economy-wide average human capital is ambiguous. Under certain conditions, it may increase.*

Proof. This result should be obvious given what we seen done so far. If bad school quality increases drastically, it is possible for the L people's human capital to increase enough to outweigh the fall in human capital experienced by the H people. However, it is also possible for economy-wide capital to reduce if bad school quality increases only moderately or decreases. □

4.3.6 Other Cases

Thus far we have analyzed the case where $\max S(q_j; H, g_H^*) \geq \max S(q_j; L, g_L^*)$ and $q_G' \geq q_B'$. We now move to a brief description of the outcomes in the other cases.

Case 2: $\max S(q_j; H, g_H^*) \geq \max S(q_j; L, g_L^*)$ **and** $q_G' \leq q_B'$

In this case, following the method of analysis we have used previously, it is easy to see that the first λn schools will choose q_G' in hope of attracting an H candidate and the rest will choose q'_B. However, since $q'_G \leq q'_B$, the applicants will accept offers from the schools that have q'_B first. Moreover, at q'_B, the expected payoff from a low-caste applicant is higher than that from a high-caste applicant, leading to these schools ranking L students above H students. So, all H students go to schools with quality q'_G and all L students go to schools with quality q'_B.

This result is perverse, in that low-caste people end up going to better quality schools than their high-caste counterparts, which is against the real world experience. So, even though it may theoretically be possible for this to happen, the exact specifications of the distribution functions and the human capital production functions it would require to happen would have to be unrealistic.

Case 3: $\max S(q_j; H, g_H^*) \leq \max S(q_j; L, g_L^*)$ **and** $q''_G \leq q''_B$

Here, the first $(1-\lambda)n$ schools would choose q''_B in a bid to attract an L student and the rest would choose q''_G. All L students would go to schools with quality q''_B and all H students would go to schools with quality q''_G.

Again, this implies that L students go to better quality schools.

Case 4: $\max S(q_j; H, g_H^*) \leq \max S(q_j; L, g_L^*)$ **and** $q''_G \geq q''_B$

Here again, the first $(1-\lambda)n$ schools would choose q''_B in a bid to attract an L student and the rest would choose q''_G. However, at q''_G, the expected payoff from an H candidate is higher than that from an L candidate. Also, applicants will accept offers from schools with quality q''_G first since $q''_G \geq q''_B$. All H students end up going to schools with quality q''_G and all L students to schools with quality q''_B.

In this case, H students go to better quality schools, but what is strange is that the schools that enter first are competing for L students. This case may not be as implausible as the previous two, and the remaining analysis is fairly similar to what we have done earlier, except that it is no longer certain that the better quality schools are of a lower quality with affirmative action. Also, whereas without affirmative action it is the first λn schools that become high quality, with affirmative action it is the last λn that become high quality.

4.4 Summary of Main Results

In this chapter, we introduced schools as economic agents by allowing them to choose their quality. We argued that schools care about the eventual human capital attained by their students, and so indirectly about whether they made it to college. We also assumed that schools could only view the caste of potential applicants. We found that the initial schools to enter bid quality up to a very high level in order to attract high-caste students, while the entrants towards the end settled for low-caste students and chose a low quality level. As a result, we again had the high-caste grade distribution first order stochastically dominating the low-caste distribution. Realized human capital was also unequal, resulting in this model from the statistical distribution practiced by schools in addition to high-caste parents being generally richer and the high-caste over-representation at college.

With affirmative action, we argued that good quality schools would always reduce quality as high-caste students become less attractive. The incentives facing bad quality schools were more complicated, and we concluded that these schools would change quality in the direction that led to more people being placed in the vicinity of the cut-off. Thus, we observed that affirmative action could lead to a universal decline in school quality. We also showed that affirmative action did not erase inequality, and we noted the possibility of an increase in inequality under certain circumstances.

Chapter 5

Effort Choice

Thus far we have treated the objects of affirmative action, i.e. the students, as inanimate. Here, we recognize students as economic agents who choose how hard to work given an admissions regime. Although the question of effort incentives has not been a part of the public debate on affirmative action, the issue is important, as it could have crucial effects on human capital levels, and hence have implications for inequality. Here, we build a model to illustrate how effort levels are chosen by students, and what the effect of affirmative action on that choice is. As our outcome variables, we shall focus on, in addition to the human capital variables, effort and welfare.

5.1 Model Set Up

5.1.1 School Education, College Education and Individual Characteristics

We retain the same definitions of $g(q, a, p, e)$, $k(\mu, g)$ and $H(a, p)$ used in Chapter 3, obeying Assumptions 3.1, 3.3 and 3.4. However, we now abstract from variations in school quality and allow e to vary, so we have the following simplifying assumption.

Assumption 5.1. *q is non-stochastic and constant across the population.*

We also make the following separability assumption on $g(q, a, p, e)$.

Assumption 5.2. *a and p enter into $g(q, a, p, e)$ in a way that allows us to define a variable $\tau = t(a, p)$ increasing in both its arguments such that $g(q, a, p, e) = \tilde{g}(q, t(a, p), e)$.*

As a simple example, if we have $g = qape$, then we would have $\tau = ap$ and $\tilde{g} = q\tau e$. τ may be viewed as a measure of a student's 'educability'. We do this to condense the variation of individual characteristics into one variable, which will allow us to draw two-dimensional diagrams. This assumption is not crucial, but without it we would have to draw three-dimensional diagrams, and since diagrammatic analysis is central to this model, it is felt that simple two-dimensional diagrams are essential to clearly convey ideas. Note, it is important (though non-crucial) that we specify that τ is increasing in its arguments so as to ensure that $\tilde{g}$ is increasing in τ and hence simplify the consequent analysis.

5.1.2 Students as Agents

In this model, the focus will be on effort choice, and thus the most interesting agents are the students. We assume that students choose an effort level e.

Assumption 5.3. *Costs of effort*[1] *are continuous, twice differentiable and homogeneous across students at* $C(e)$ *with* $C(0) = 0$, $C'(e) > 0$ *and* $C''(e) > \max\left\{\frac{\partial^2 \tilde{g}}{\partial e^2}\ ,\ \frac{\partial k}{\partial \tilde{g}}\frac{\partial^2 \tilde{g}}{\partial e^2} + \left(\frac{\partial \tilde{g}}{\partial e}\right)^2 \frac{\partial^2 k}{\partial \tilde{g}^2}\right\}$.

The last condition is simply that costs are more convex than both $\tilde{g}$ and k, which is required to ensure an interior solution for effort.

Regarding payoffs to the students, we assume that the benefit to the students is simply the final level of human capital they end up with. We can thus define a payoff function $S(e;\tau)$ for the students[2] where

$$S(e;\tau) = \begin{cases} \tilde{g}(q,\tau,e) - C(e) & \text{if student doesn't go to college} \\ k(\mu,\tilde{g}(q,\tau,e)) - C(e) & \text{if student goes to college} \end{cases} \tag{5.1}$$

Note, the students are assumed to know the value of τ and can observe the value of q, so they know the value of their payoff with certainty; we need not get into expectations as in Chapter 4.

5.1.3 Sequence of Actions

The sequence of actions is as follows.

1. n people are born, with nature assigning each person a caste, H with probability λ and L with probability $1-\lambda$.

2. Children are admitted to school.

[1] This function should not be confused with the cost of investment in school quality function in Chapter 4.
[2] This should, again, not be confused with the schools' payoff function in Chapter 4.

3. Students choose an effort level e at cost $C(e)$.
4. Students undergo education and attain grades.
5. Colleges admit students based on the mechanical cut-off rule.
6. Those admitted undergo college education.
7. Payoffs are received.

Here, we end up with only one interesting decision node to analyze, namely the students' choice of effort.

5.2 Results Without Affirmative Action

5.2.1 Results on τ

Lemma 5.1. *$\tilde{g}(q, \tau, e)$ is increasing in τ, and all cross derivatives are positive.*

Proof. We know that

$$g(q, a, p, e) = \tilde{g}(q, \tau, e) \tag{5.2}$$

where $\tau = t(a, p)$. Differentiating with respect to a, we get

$$\frac{\partial g}{\partial a} = \frac{\partial \tilde{g}}{\partial \tau}\frac{\partial \tau}{\partial a} \tag{5.3}$$

We know that $\frac{\partial g}{\partial a} > 0$ from Assumption 3.1. We also specified that τ is increasing in its arguments, so we know $\frac{\partial \tau}{\partial a} > 0$. Thus, we must have $\frac{\partial \tilde{g}}{\partial \tau} > 0$. This proves the first claim.

To prove the second claim, we differentiate equation 5.3 with respect to e to get

$$\frac{\partial^2 g}{\partial e \partial a} = \frac{\partial \tau}{\partial a}\frac{\partial^2 \tilde{g}}{\partial e \partial \tau} \tag{5.4}$$

Again, we know from Assumption 3.1 that $\frac{\partial^2 g}{\partial e \partial a} > 0$, so we must have $\frac{\partial^2 \tilde{g}}{\partial e \partial \tau} > 0$. We can similarly prove that $\frac{\partial^2 \tilde{g}}{\partial q \partial \tau} > 0$. To see that $\frac{\partial^2 \tilde{g}}{\partial e \partial q} > 0$, we simply differentiate equation 5.2 with respect to both e and q. □

We now consider results on the distribution of τ.

Lemma 5.2. *τ for caste i is distributed according to the distribution function $F_i(\tau) = E_a\left[B_i(P(a;\tau))\right]$ where $P(a, \tau)$ is implicitly defined by $\tau = t(a, P(a;\tau))$.*

Proof. Analogous to that of Lemma 3.1, except that we take $\tau = t(a,p)$ as the starting point instead of the grades function. However, we do appeal to the grades equation to show that $P(a,\tau)$ is a decreasing function. □

We now move on to a first order stochastic dominance result for τ.

Theorem 5.1. $F_H(\tau) \leq F_L(\tau)\ \forall \tau$

Proof. Analogous to that of Theorem 3.1, with g being replaced by τ. □

5.2.2 Solving for Optimal Effort

Solving for the optimal e for every value of τ is not a simple optimization exercise, for the choice of e may determine whether the student goes to college or not and hence alters the functional form of the student's objective function. This discontinuity arises due to rationing of college seats and has important effects for decision making, as we shall see. It is arguably an extremely important feature of most educational systems around the world.

One way to deal with the discontinuity in the student's payoff function is as follows. We first find the optimal e while imposing the constraint that the student must make it to college, so that we can work with a single functional form of the objective function $k(\mu, \tilde{g}(q,\tau,e)) - C(e)$. Then we carry out the same exercise while imposing the constraint that the student does not make it to college, so that we can work with the functional form $\tilde{g}(q,\tau,e) - C(e)$ of the objective function. This will give us two choices of e for each τ, one corresponding to not getting into college and the other to getting into college. We can then compare the resultant payoffs and choose the e that corresponds to the higher payoff.

Step 1: Obtaining Two e's for Each τ

Lemma 5.3. *The optimal effort schedule constrained by making it to college is given by* $e_C(\tau)$ *which is defined by*

$$\begin{array}{ll} \tilde{g}(q,\tau,e_C(\tau)) = g^* & \textit{when } \tau \leq \tau_0 \\ \frac{\partial k}{\partial \tilde{g}} \frac{\partial \tilde{g}(q,\tau,e_C(\tau))}{\partial e} - C'(e_C(\tau)) = 0 & \textit{when } \tau \geq \tau_0 \end{array} \tag{5.5}$$

where τ_0 *is defined by the intersection of the two functions in 5.5.*

The optimal effort schedule constrained by not making it to college is given by $e_N(\tau)$ *which* *is defined by*

$$\begin{aligned} &\tfrac{\partial \tilde{g}(q,\tau,e_N(\tau))}{\partial e} - C'(e_N(\tau)) = 0 \quad && \textit{when } \tau \leq \tau_1 \\ &\tilde{g}(q,\tau,e_N(\tau)) = g^* && \textit{when } \tau \geq \tau_1 \end{aligned} \tag{5.6}$$

where τ_1 *is defined by the intersection of the two functions in 5.6.*

Proof. First note that, given a cut-off grade g^*, in order to get into college, a student would need to score at least g^*. Similarly, to not make it, we must restrict the student to grades below g^*. The equation

$$\tilde{g}(q,\tau,e) = g^* \tag{5.7}$$

implicitly defines e as a function of τ, and gives the locus of all $e - \tau$ combinations that correspond to achieving the cut-off grade. This will be a decreasing function, as $\tilde{g}$ is increasing in all its arguments. This can also be seen by implicitly differentiating treating e as a function of τ to give

$$\frac{\partial e}{\partial \tau} = -\frac{\frac{\partial \tilde{g}}{\partial \tau}}{\frac{\partial \tilde{g}}{\partial e}} < 0 \tag{5.8}$$

To the above and right of this function, the student scores more than g^*; and below and to the left of this function, the student scores less than g^*. This function then divides the space into two parts, one where the student makes it to college and one where he doesn't. Call this function GG

Next, we maximize each objective function. First, given the 'not making it to college' objective function, the optimal choice of effort satisfies

$$\underset{e}{arg\,max}\,\{\tilde{g}(q,\tau,e) - C(e)\} \Rightarrow \frac{\partial \tilde{g}(q,\tau,e)}{\partial e} - C'(e) = 0 \tag{5.9}$$

This again implicitly defines e as a function of τ. To see what this function looks like, we note that at $\tau = 0$, we must necessarily have $e = 0$, as $\tilde{g}(q,0,e) = 0$ from Assumption 3.1 and so the only way to maximize the objective function is to minimize costs by choosing $e = 0$. Also, the function is upward sloping, which can be seen by implicit differentiation to get

$$\frac{\partial e}{\partial \tau} = \frac{\frac{\partial^2 \tilde{g}}{\partial e \partial \tau}}{C''(e) - \frac{\partial^2 \tilde{g}}{\partial e^2}} > 0 \tag{5.10}$$

This inequality holds because we know from Assumption 3.1 that all cross derivatives of $\tilde{g}$ are positive, and from Assumption 5.3 that costs increase faster than human capital, which in turn increases faster than grades by definition. Call this function NN.

Next, we repeat this exercise with the 'making it to college' objective function. The optimal choice of effort satisfies

$$arg\max_{e} \{k(\mu, \tilde{g}(q, \tau, e)) - C(e)\} \Rightarrow \frac{\partial k}{\partial \tilde{g}} \frac{\partial \tilde{g}(q, \tau, e)}{\partial e} - C'(e) = 0 \quad (5.11)$$

This again implicitly defines e as a function of τ, and starts at the origin for the same reason as above. This will also always lie above NN because we know from Assumption 3.3 that $\frac{\partial k}{\partial \tilde{g}} \geq 1$, which means that for any given τ, the optimal choice of effort be higher than in NN. The slope of this function is ambiguous, as implicit differentiation gives

$$\frac{\partial e}{\partial \tau} = \frac{\frac{\partial k}{\partial \tilde{g}} \frac{\partial^2 \tilde{g}}{\partial \tau \partial e} + \frac{\partial^2 k}{\partial \tilde{g}^2} \frac{\partial \tilde{g}}{\partial e} \frac{\partial \tilde{g}}{\partial \tau}}{C''(e) - \frac{\partial k}{\partial \tilde{g}} \frac{\partial^2 \tilde{g}}{\partial e^2} - \left(\frac{\partial \tilde{g}}{\partial e}\right)^2 \frac{\partial^2 k}{\partial \tilde{g}^2}} \quad (5.12)$$

We know that the denominator is positive from Assumption 5.3, and that all the terms in the numerator save $\frac{\partial^2 k}{\partial g^2}$ are positive from Assumptions 3.1 and 3.3. This means that unless k is extremely concave in g, the slope will be positive. In drawing the function as positively sloped, we are implicitly assuming that k is not too concave in g. Indeed, given that k must always increase faster than grades, it is difficult to see how it is possible for k to be extremely concave. Call this function CC.

We can graph GG, NN and CC to get Figure 5.1. We can now carry out our analysis as outlined earlier. For the student to go to college, he must be above and to the right of GG. For $\tau \geq \tau_0$, the constraint of getting into college is not binding and the optimal choice of effort is given by CC. However, for $\tau \leq \tau_0$ the constraint is binding. Our assumptions on $C(e)$ imply $\frac{\partial^2 S(e;\tau)}{\partial e^2} < 0$, which means that payoff decreases the further away we get from CC for any given τ. Thus, best the student can do is to choose a level of effort that just gets him into college. This gives us the optimal effort schedule constrained by getting into college $e_C(\tau)$, which is shown by a thick line. A similar logic will give us $e_N(\tau)$. □

Step 2: Comparing Payoffs at Each e to get the Optimal Payoff Schedule

Lemma 5.4. *The optimal payoff schedule is given by*

$$\hat{S}(\tau) = \begin{cases} S(e_N(\tau), \tau) = \tilde{g}(q, \tau, e_N(\tau)) - C(e_N(\tau)) & when\ \tau \leq \tau^* \\ S(e_C(\tau), \tau) = k(\mu, \tilde{g}(q, \tau, e_C(\tau))) - C(e_C(\tau)) & when\ \tau \geq \tau^* \end{cases} \quad (5.13)$$

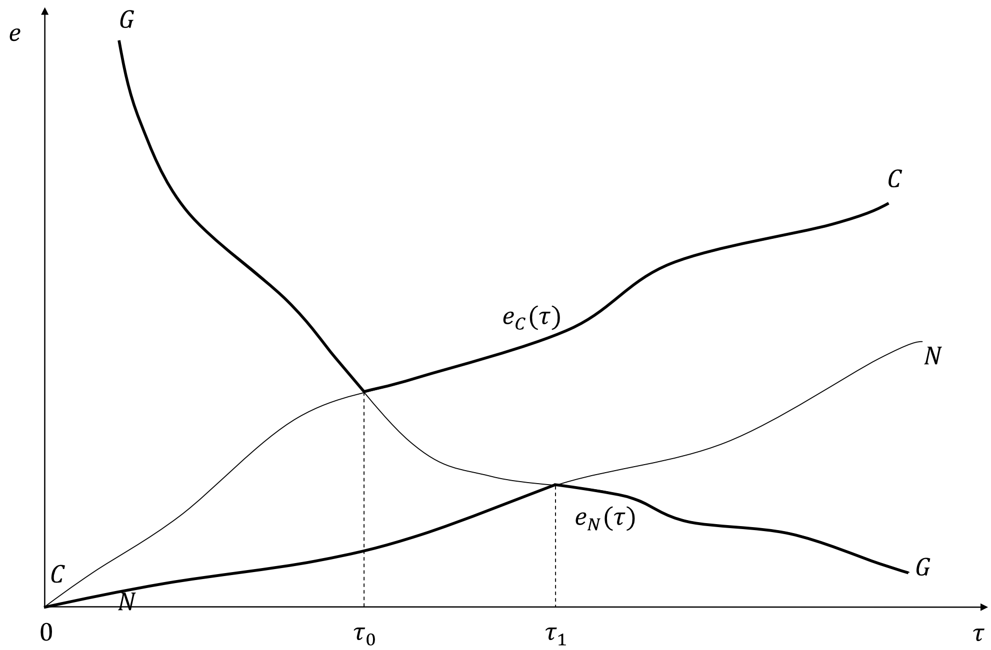


Figure 5.1: Obtaining Two e's for Each τ

where τ^* is defined by the intersection of the two functions in 5.13.

Proof. In order to compare the payoffs corresponding to the two constrained optimal effort schedules we must first find out what they look like. We start with the payoff schedule corresponding to $e_N(\tau)$, i.e. the case when the student is constrained to not attending college. In this case, the schedule is given by

$$\hat{S}_N(\tau) = S(e_N(\tau); \tau) = \tilde{g}(q, \tau, e_N(\tau)) - C(e_N(\tau)) \tag{5.14}$$

We know that at $\tau = 0$, $\tilde{g} = 0$ and $e = 0$, so $\hat{S}_N(0) = 0$. Next, we know that up to τ_1, $e_N(\tau)$ satisfies the first order condition in equation 5.9. The slope of $\hat{S}_N(\tau)$ up to τ_1 is thus given by

$$\left.\frac{\partial \hat{S}_N(\tau)}{\partial \tau}\right|_{\tau \leq \tau_1} = \frac{\partial \tilde{g}}{\partial \tau} + \left[\frac{\partial \tilde{g}}{\partial e} - C'(e)\right] \frac{\partial e}{\partial \tau} = \frac{\partial \tilde{g}}{\partial \tau} > 0 \tag{5.15}$$

Beyond τ_1, $e_N(\tau)$ has a slope given by equation 5.8. Using this, the slope of $\hat{S}_N(\tau)$ beyond τ_1 is given by

$$\left.\frac{\partial \hat{S}_N(\tau)}{\partial \tau}\right|_{\tau \geq \tau_1} = \frac{\partial \tilde{g}}{\partial \tau} + \frac{\partial \tilde{g}}{\partial e}\left(-\frac{\frac{\partial \tilde{g}}{\partial \tau}}{\frac{\partial \tilde{g}}{\partial e}}\right) - C'(e)\left(-\frac{\frac{\partial \tilde{g}}{\partial \tau}}{\frac{\partial \tilde{g}}{\partial e}}\right) = C'(e)\frac{\frac{\partial \tilde{g}}{\partial \tau}}{\frac{\partial \tilde{g}}{\partial e}} > 0 \tag{5.16}$$

Also note that $\hat{S}_N(\tau)$ is continuous since $\tilde{g}$ and C are continuous and $e_N(\tau)$ is continuous (even though it is not differentiable everywhere). Thus, $\hat{S}_N(\tau)$ starts at the origin and has a positive slope throughout.

Next, we move to the payoff schedule corresponding to $e_C(\tau)$, i.e. the case when the student is constrained to attending college. In this case, the schedule is

$$\hat{S}_C(\tau) = S(e_C(\tau);\tau) = k(\mu, \tilde{g}(q, \tau, e_C(\tau))) - C(e_C(\tau)) \tag{5.17}$$

Here, at $\tau = 0$, $\tilde{g} = 0$ but $e \to \infty$, so $S_C(0) \to -\infty$. Next, in a manner analogous to above we can show that

$$\begin{aligned} \left.\frac{\partial \hat{S}_C(\tau)}{\partial \tau}\right|_{\tau \leq \tau_0} &= C'(e)\frac{\frac{\partial \tilde{g}}{\partial \tau}}{\frac{\partial \tilde{g}}{\partial e}} > 0 \\ \left.\frac{\partial \hat{S}_C(\tau)}{\partial \tau}\right|_{\tau \geq \tau_0} &= \frac{\partial k}{\partial \tilde{g}}\frac{\partial \tilde{g}}{\partial t} > 0 \end{aligned} \tag{5.18}$$

Also, we note that for $\tau \geq \tau_0$, $\hat{S}_C(\tau) > \hat{S}_N(\tau)$. This can be seen as follows. Between τ_0 and τ_1, both schedules correspond to unconstrained maxima. However, given any τ, the unconstrained maximum of $k(\mu, \tilde{g}(q, \tau, e)) - C(e)$ will always be greater than the unconstrained maximum of $\tilde{g}(q, \tau, e) - C(e)$. This is because $k > \tilde{g}$ at every τ from Assumption 3.3. Beyond τ_1, $\hat{S}_N(\tau)$ corresponds to the constrained maxima, which will always be less than the unconstrained maxima, which in any case is less than $\hat{S}_C(\tau)$. Finally, we note that $\hat{S}_C(\tau)$ is continuous as well. So, we have established that $\hat{S}_C(\tau)$ begins at $-\infty$, rises throughout, and beyond τ_0 must be greater than $\hat{S}_N(\tau)$. This means that $\hat{S}_C(\tau)$ must cross $\hat{S}_N(\tau)$ only once, and the crossing must be at some $\tau^* < \tau_0$.

Having established this, we can draw Figure 5.2. Now, we are in a position to compare payoffs. We have established that before τ^*, a student gets a higher payoff not going to college, and after τ^* by going to college. Therefore, all students with $\tau < \tau^*$ will not go to college while students with $\tau \geq \tau^*$ will go to college. $\square$

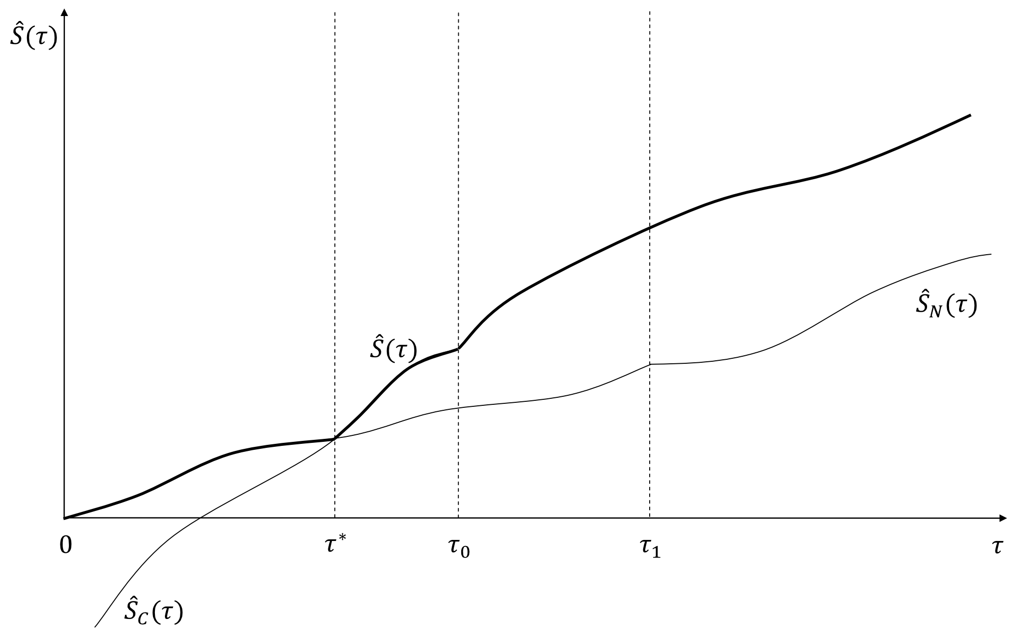


Figure 5.2: Comparing Payoffs to get the Optimal Payoff Schedule

Step 3: The Optimal Effort Schedule

Theorem 5.2. *The optimal effort schedule is given by* $e(\tau)$*, which satisfies*

$$\begin{array}{ll} \frac{\partial \tilde{g}(q,\tau,e(\tau))}{\partial e} - C'(e(\tau)) & \text{when } \tau < \tau^* \\ \tilde{g}(q,\tau,e(\tau)) = g^* & \text{when } \tau^* \leq \tau \leq \tau_0 \\ \frac{\partial k}{\partial \tilde{g}} \frac{\partial \tilde{g}(q,\tau,e(\tau))}{\partial e} - C'(e(\tau)) & \text{when } \tau \geq \tau_0 \end{array} \tag{5.19}$$

Proof. From Lemma 5.4, we know that students with $\tau < \tau^*$ will go to school, so the optimal effort before τ^* is given by $e_N(\tau)$. Similarly, students with $\tau \geq \tau^*$ will go to college, so over this range the optimal effort is given by $e_C(\tau)$. This is displayed in Figure 5.3. □

Figure 5.3 has the following intuitive interpretation. For people with extremely low levels of τ (i.e., $\tau < \tau^*$), it takes too much effort to get into college and the disutility of putting that effort in is greater than the extra returns. So they choose to 'take it easy', as it were, and do not work hard enough to get into college. People with extremely high levels of τ (i.e. $\tau \geq \tau_0$) also need not put any 'extra' effort in, as they will get into college without trying harder than they have to. For these two sets of people, the only incentive to work is to get most out of school and college respectively. The fact that going to college increases returns

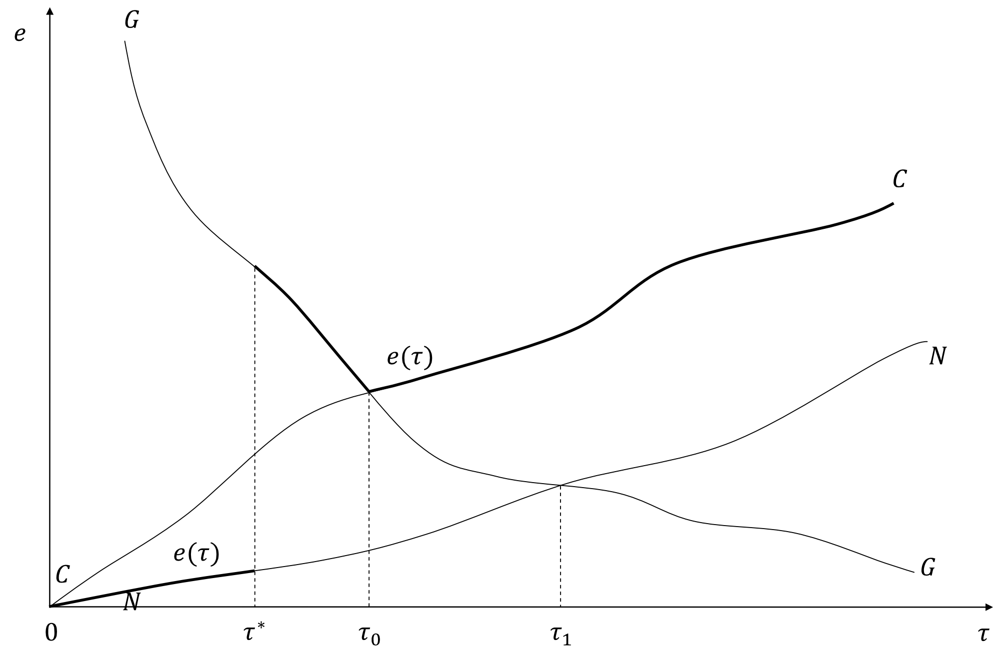


Figure 5.3: The Optimal Effort Schedule

is not a binding factor in either of these cases, and so does not provide any extra incentive. For people in the mid-range (i.e. $\tau^* \leq \tau \leq \tau_0$), there is an added incentive to work hard, as working harder means that they get into college and enjoy higher returns. These students have a corner optimum because getting in to college is a binding constraint.

5.2.3 The Cut-Off Grade

From the preceding analysis, we know that all students with $\tau \geq \tau^*$ must go to college. However, the admissions constraint must also be satisfied. Therefore, τ^* must be determined by

$$[(1 - F_H(\tau^*))\,\lambda + (1 - F_L(\tau^*))\,(1 - \lambda)]\,n = \bar{A} \tag{5.20}$$

Once τ^* is determined, we must find a g^* that is consistent with it. This can be done as follows. τ^* is determined by the crossing of the two constrained maximum payoff schedules. We know that this crossing takes place before τ_0, where $\hat{S}_C(\tau)$ depends on g^*. Thus, having determined τ^*, we only need to find a g^* so that $\hat{S}_C(\tau^*) = \hat{S}_N(\tau^*)$.

An alternate method of calculating the cut-off grade would be to first determine the distribution of grades and then directly determine g^*, but this way is completely equivalent to the one described as the distribution of grades would depend critically on the distribution of τ.

Theorem 5.3. *In the effort choice model, without affirmative action, the H community is over-represented at the college level.*

Proof. From Theorem 5.1, $(1 - F_H(\tau^*)) \geq (1 - F_L(\tau^*))$. $\square$

5.2.4 Realized Human Capital

We shall first determine the level of realized human capital for each level of τ. We proceed as follows. For $\tau < \tau^*$, the student does not go to college, so the realized human capital is just $\tilde{g}(q, \tau, e(\tau))$. We know that over this range, $e(\tau)$ is increasing, so as τ increases, $\tilde{g}(q, \tau, e(\tau))$ increases as well. For $\tau* \leq \tau \leq \tau_0$, we know that the student goes to college, and so realized human capital is $k(\mu, \tilde{g}(q, \tau, e(\tau)))$. However, we know that in this range $e(\tau)$ is decreasing so that grades are constant at $\tilde{g}(q, \tau, e(\tau)) = g^*$. Hence, in this range, realized human capital is also constant at $k(\mu, g^*)$. Finally, for $\tau \geq \tau_0$, we again have the students going to college, but now $e(\tau)$ is increasing, and so human capital is also increasing. We also note that $\kappa(\tau)$ must be continuous everywhere except at τ^*, where there will be an upward jump. This is because $e(\tau)$ is continuous everywhere except at τ^*, where there is a positive jump. We can thus describe the realized human capital schedule by

$$\kappa(\tau) = \begin{cases} \tilde{g}(q, \tau, e(\tau)) & \text{when } \tau < \tau^* \\ k(\mu, \tilde{g}(q, \tau, e(\tau))) = k(\mu, g^*) & \text{when } \tau^* \leq \tau \leq \tau_0 \\ k(\mu, \tilde{g}(q, \tau, e(\tau))) & \text{when } \tau \geq \tau_0 \end{cases} \tag{5.21}$$

Note that the realized human capital schedule always has a non-negative slope. $\kappa(\tau)$ can be displayed diagrammatically as in Figure 5.4.

Average human capital for caste i is then given by

$$\tilde{k}_i = \int_0^\infty \kappa(\tau)\, dF_i(\tau) \tag{5.22}$$

Lemma 5.5. *The distribution of realized human capital is unequal, with $\tilde{k}_H \geq \tilde{k}_L$.*

Proof. Integrating by parts, we get

$$\tilde{k}_i = \int_0^\infty \kappa'(\tau)\, (1 - F_i(\tau))\; d\tau \tag{5.23}$$

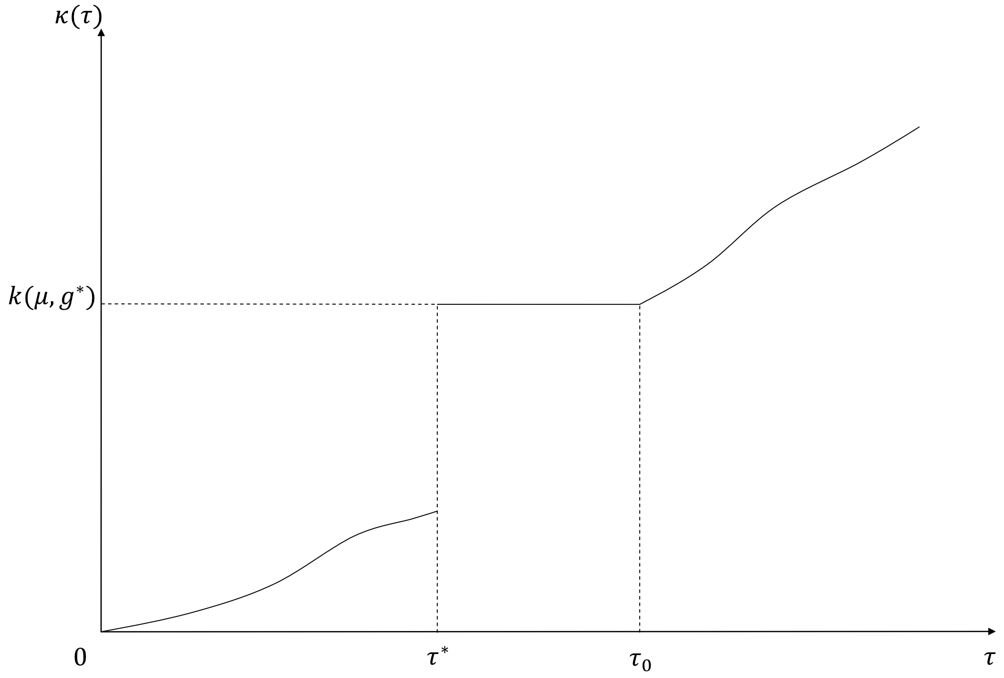


Figure 5.4: Realized Human Capital

Now, we know that $\kappa'(\tau)$ is non-negative and $1 - F_H(\tau) \geq 1 - F_L(\tau)$ from Theorem 5.1, and so the result follows. □

5.2.5 Students' Welfare

The realized welfare for students is described by $\hat{S}(\tau)$ as in Lemma 5.4, and can be drawn as in Figure 5.2. We again note that $\hat{S}'(\tau)$ is non-negative.

Average welfare for caste i is given by

$$\tilde{S}_i = \int_0^\infty \hat{S}(\tau)\, dF_i(\tau) \tag{5.24}$$

Lemma 5.6. *The distribution of realized welfare is unequal, with $\tilde{S}_H \geq \tilde{S}_L$.*

Proof. Analogous to that of Lemma 5.5. □

5.3 Affirmative Action

Equal representation implies the choosing of two cut-off grades g_H^* and g_L^* such that the same proportion of each caste gets into college. Let us first look at what changes a new cut-off grade implies for optimal effort.

5.3.1 Solving for Optimal Effort

One can solve for optimal effort in the same way as earlier. The only difference with affirmative action is that the generic cut-off grade g^* is replaced by g_i^* for caste i. Thus, we can go through the same three steps for caste i, using g_i^* in place of g^*. We will obtain $e_{Ci}(\tau)$ and $e_{Ni}(\tau)$ schedules in Step 1; a $\hat{S}_i(\tau)$ schedule in Step 2; and an $e_i(\tau)$ schedule in step three with functional forms exactly the same as given in Lemmas 5.3 and 5.4 and Theorem 5.2 except that g^* will be replaced by g_i^*. We can also determine τ_{0i}, τ_{1i} and τ_i^* just as we determined τ_0, τ_1 and τ^* before.

We now determine how the values of τ_{0i}, τ_{1i} and τ_i^* move in relation to τ_0, τ_1 and τ^* as g^* moves to g_i^*.

Lemma 5.7. *$\tau_{0i} > \tau_0$ and $\tau_{1i} > \tau_1$ if $g_i^* > g^*$ and vice versa.*

Proof. The effects of an increase in the cut-off grade are illustrated in Figure 5.5. We first

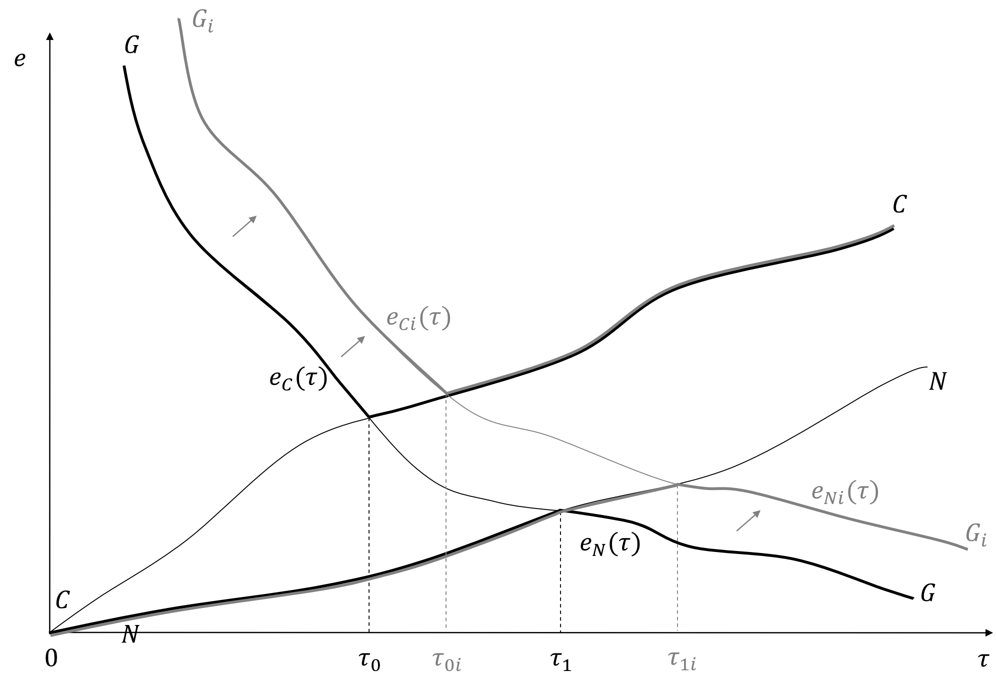


Figure 5.5: The Effect of Increasing g^*

note that an increase in g^* to g_i^* implies that GG shifts out and to the right. To see this, we note that GG has become $\tilde{g}(a, \tau, e) = g_i^*$, so for every level of τ, a higher level of e is needed as the grade that needs to be achieved has increased. Also, NN and CC remain the same, as g^* does not enter into their determination. Now it is obvious that since the negatively sloped line has moved outwards, its intersections with the positively sloped lines will also move outwards. Similarly, one can show that if g^* falls, τ_0 and τ_1 will reduce. □

Lemma 5.8. *$\tau_i^* > \tau^*$ if $g_I^* > g^*$ and vice versa.*

Proof. The effects of increasing the cut-off grade on the payoff schedules are depicted in Figure 5.6.

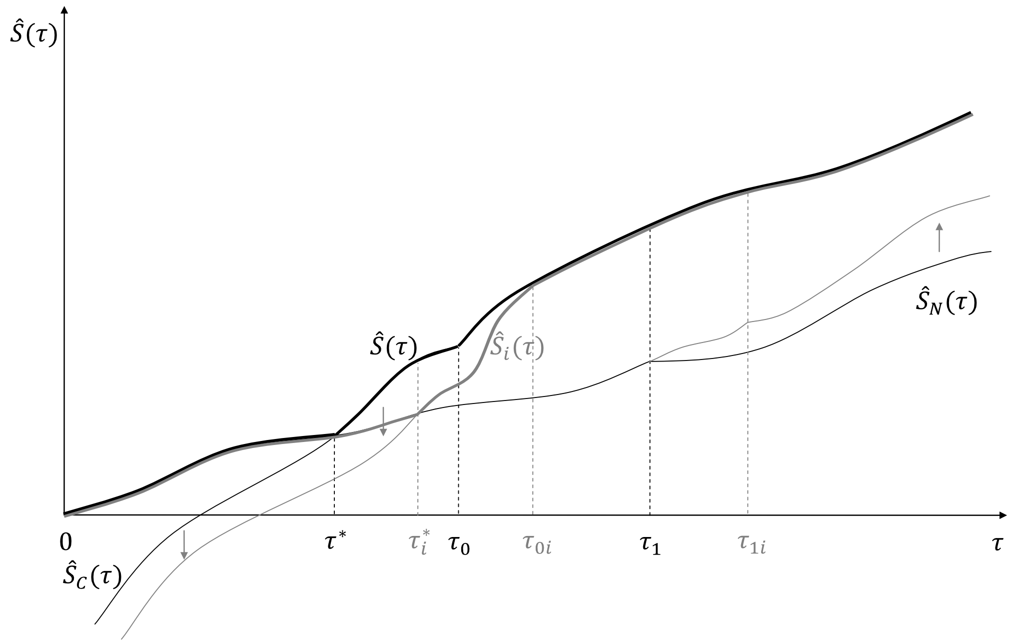


Figure 5.6: The Effect of Increasing g^* on the Payoff Schedule

We first look at what happens to $\hat{S}_N(\tau)$ and $\hat{S}_C(\tau)$ as g^* changes. Looking first at $\hat{S}_{Ni}(\tau)$, we notice that g^* enters into its determination only after τ_{1i}. Given the new effort schedule $e_{Ni}(\tau)$ we just obtained while proving Lemma 5.7, we can see that if $g_i^* > g^*$, the effort levels in this range increase, and move closer to the optimum. Remember that $\tilde{g}(q, \tau, e) - C(e)$ is concave in e, and so move closer to the optimum must raise welfare in this range. However, as this is still less than the unconstrained maximum, which itself is less than the unconstrained maximum of $k(\mu, \tilde{g}(q, \tau, e)) - C(e)$ at each point, the increase will not be enough to rise above $\hat{S}_C(\tau)$. Also note that since τ_0 has moved to τ_{0i}, $\hat{S}_{Ni}(\tau)$ contains an additional extension of the original pre-τ_0 section so that we get a continuous curve.

Now, we look at $\hat{S}_{Ci}(\tau)$. Here, g^* enters into the determination of the function before τ_{0i}. Again, given the new effort schedule $e_{Ci}(\tau)$ obtained in the proof of Lemma 5.7, we know that over this range, effort has increased, and moved further away from the optimum, which will reduce payoff as we know that $k(\mu, g(q, \tau, e)) - C(e)$ is concave in e. Again, as τ^* has risen to τ_H^*, $\hat{S}_{Ci}(\tau)$ does not contain the section of $\hat{S}_C(\tau)$ between τ^* and τ_H^*.

Since the intersection of $\hat{S}_{Ci}(\tau)$ and $\hat{S}_{Ni}(\tau)$ must take place before τ_{0i}, and the section of $\hat{S}_{Ci}(\tau)$ corresponding to this range (the steeper positively sloped curve) has moved down while that of $\hat{S}_{Ni}(\tau)$ has not shifted (the flatter positively sloped curve), the intersection must be at a higher point, i.e. $\tau_i^* > \tau^*$.

It can similarly be shown that if $g_i^* < g^*$, then $\tau_i^* < \tau^*$. □

5.3.2 The Cut-Off Grades

Lemma 5.9. *In the effort choice model, affirmative action leads to a rise in the cut-off grade for the high-caste and a fall for the low-caste.*

Proof. As before, we know that everyone from caste i with $\tau \geq \tau_i^*$ will go to college. This means that a proportion $1 - F_i(\tau_i^*)$ of each caste will go to college. The only way then to achieve equal representation would be to set

$$1 - F_H(\tau_H^*) = 1 - F_L(\tau_L^*) \tag{5.25}$$

This equal representation condition along with the admissions constraint

$$\left[(1 - F_H(\tau_H^*))\,\lambda + (1 - F_L(\tau_L^*))\,(1 - \lambda)\right] n = \bar{A} \tag{5.26}$$

will determine τ_H^* and τ_L^*. Analogously to the proof of Lemma 3.3, we can show that $\tau_H^* \geq \tau^* \geq \tau_L^*$.

Then we can determine the corresponding g_H^* and g_L^* in the way described previously, by finding a g_i^* for each i such that $\hat{S}_{Ci}(\tau_i^*) = \hat{S}_{Ni}(\tau_i^*)$. However, we know from Lemma 5.8 that τ^* must move in the same direction as g_i^*, which leads to our result. □

5.3.3 Effect on Effort Choice

Theorem 5.4. *For high-caste people, affirmative action leads to reduced effort only for the people who lose college seats. For others, effort either stays constant or rises. The effect on average effort is ambiguous.*

Proof. We have so far established that for the H people, the cut-off must rise. We also know that this implies the GG curve will shift in a north-easterly direction, NN and CC will not change, $\tau_H^* \geq \tau^*$ and $\tau_{0H} \geq \tau_0$. We also assume for expositional simplicity that $\tau_H^* \leq \tau_0$. This situation is depicted in Figure 5.7.

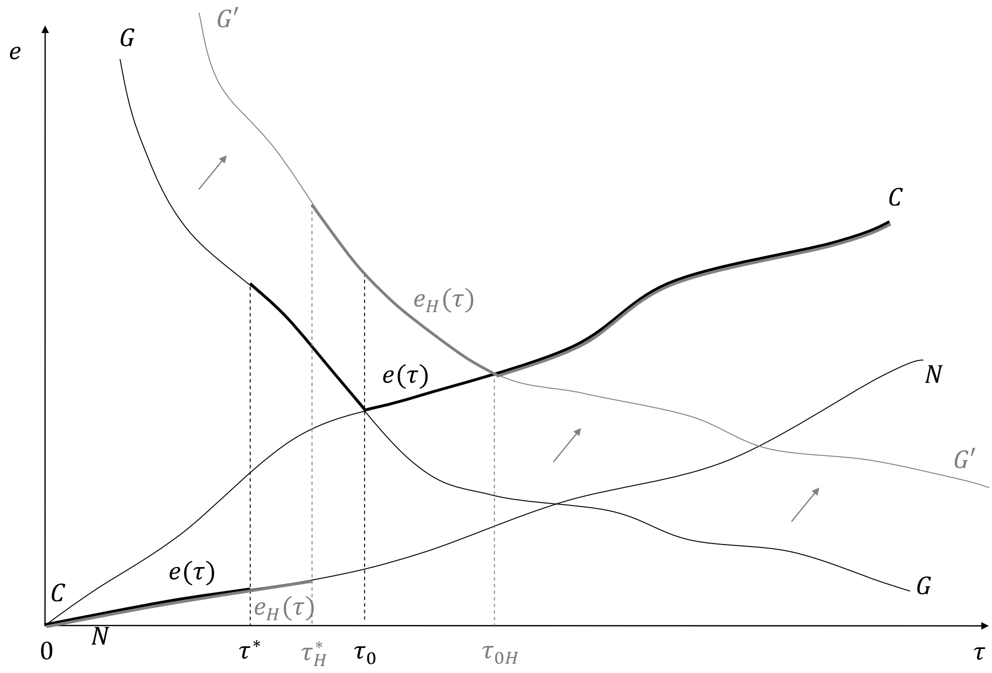


Figure 5.7: Effect on high-caste Effort

Consider people with $\tau \in [0, \tau^*)$. The effort schedules with and without affirmative action correspond exactly. For these people, without affirmative action, the extra work required to get into college and the consequent disutility meant that they got a higher payoff not trying to get into college. With affirmative action increasing the cut-off, they would require an even higher level of effort to get into college, which would clearly involve even more disutility.

People with $\tau \in [\tau^*, \tau_H^*)$ earlier found that the increased payoffs due to attending college outweighed the costs of working extra-hard to get in. Now, however, it requires more effort to get in, which makes the costs increase to outweigh benefits. These people will therefore settle for not going to college and will reduce effort, as it is now too difficult to get into college.

People with $\tau \in [\tau_H^*, \tau_0]$ found it worthwhile without affirmative action to work extra-hard to get into college. Now, getting into college takes even more effort, but even with the increased effort going to college gives them a higher payoff than settling for just a school education. So, these people increase their effort levels in a bid to get into college.

People with $\tau \in [\tau_0, \tau_{0H})$ did not earlier need to try extra-hard to get into college, but with affirmative action, the old effort levels would give them grades not good enough to get into college anymore. So, these people must increase effort levels to ensure they get into college.

Lastly, for people with extremely high values of τ, i.e. $\tau \in [\tau_{0H}, \infty)$, the grades they were attaining without affirmative action are high enough to guarantee them college seats even after affirmative action. They still do not need to bother about getting into college and therefore face no incentive to change effort levels.

As is apparent, the effect on average effort is ambiguous as it depends on the probability mass $f_H(\tau)$ that exists in each of these intervals. It is important to note, though, that affirmative action can result in a case where average effort for the high-caste increases. □

Theorem 5.5. *For low-caste people, affirmative action leads to increased effort only for people who gain college seats. For others, effort either falls or remains the same. The effect on average effort is ambiguous.*

Proof. Given earlier results, we know that the cut-off grade will fall, the GG curve will shift in a south-westerly direction, NN and CC will not change, $\tau_L^* \leq \tau^*$ and $\tau_{0L} \leq \tau_0$. Again, for expositional simplicity, we assume that $\tau_{0L} \geq \tau^*$. This situation is depicted in Figure 5.8.

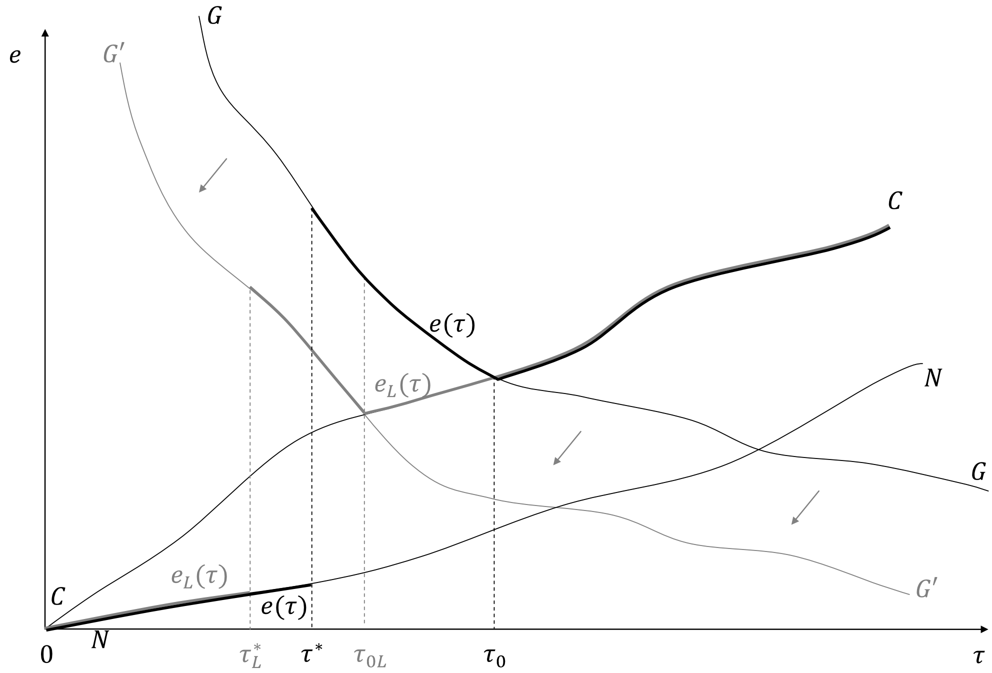


Figure 5.8: Effect on low-caste Effort

Consider people with $\tau \in [0, \tau_L^*)$. These people earlier found that the disutility from working extra-hard to get into college far outweighed the increase in human capital as a result of a college education. However, these people have a combination of parent's human capital and ability which is so low that even the reduced effort they need to put in to get into college after affirmative action is too high for them to not be content with just a school education. Effort in this range thus remains unchanged.

For people with $\tau \in [\tau_L^*, \tau^*)$ who were earlier content with just a school education, the reduction in the cut-off grade and consequently the effort cost required to get them into college means that now the increase in human capital that comes with a college education outweigh the extra costs. These people will then increase effort in order to get into college.

People with $\tau \in [\tau^*, \tau_{0L}]$ can now get into college with less work, but the reduction in the cut-off is not sufficient for the 'getting into college' constraint to stop binding. So, these people continue to work extra-hard, but less so than without affirmative action.

For people with $\tau \in [\tau_{0L}, \tau_0)$, the 'getting into college' constraint is no longer binding. They can move to their unconstrained optima and so reduce effort as they need not worry about working extra-hard just to get into college.

People with $\tau \in [\tau_0, \infty)$ did not have to worry about getting in to college before affirmative action, so if the cut-off falls, they still do not need to worry about it and therefore will continue on their unconstrained maxima.

Again, the effect on average effort depends on the distribution of τ, but it is important to note the possibility of cases where affirmative action reduces average effort for the low-caste. □

5.3.4 Realized Human Capital

We can now determine human capital for caste i in a manner analogous to the case without affirmative action to get

$$\kappa_i(\tau) = \begin{cases} \tilde{g}(q, \tau, e_i(\tau)) & \text{when } \tau < \tau_i^* \\ k(\mu, \tilde{g}(q, \tau, e_i(\tau))) = k(\mu, g_i^*) & \text{when } \tau_i^* \leq \tau \leq \tau_{0i} \\ k(\mu, \tilde{g}(q, \tau, e_i(\tau))) & \text{when } \tau \geq \tau_{0i} \end{cases} \tag{5.27}$$

Lemma 5.10. *For the high-caste, realized human capital falls only for people who have lost college seats due to affirmative action. For the others, human capital either rises or stays constant.*

Proof. Given that $\tau_H^* \geq \tau^*$ and $\tau_{0H} \geq \tau_0$, and assuming for expositional simplicity that $\tau_H^* \leq \tau_0$, we can draw Figure 5.9.

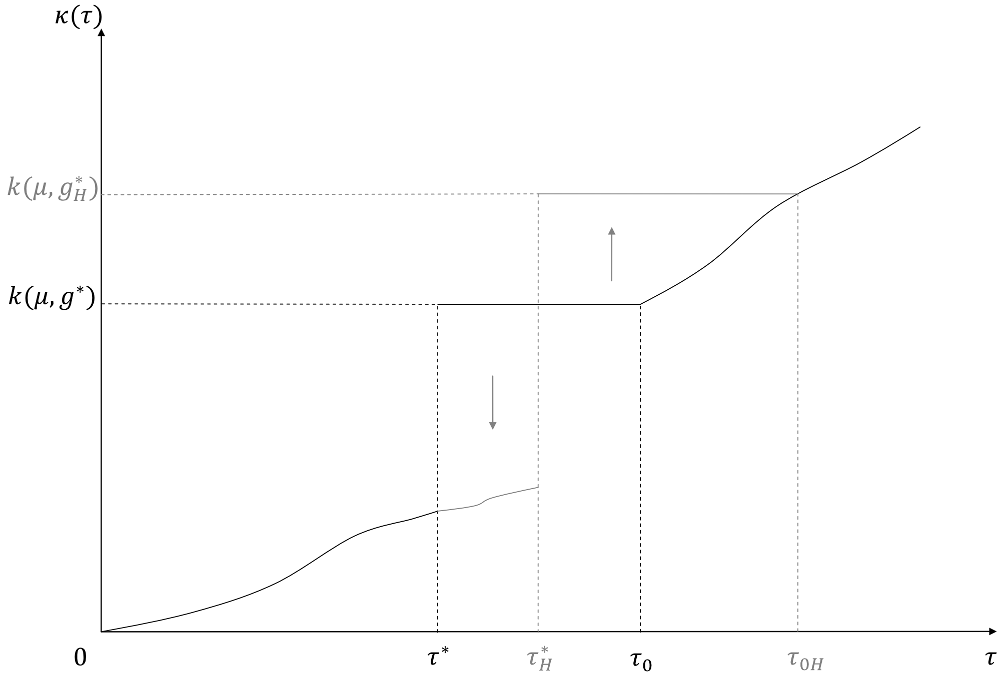


Figure 5.9: Effect on high-caste Human Capital

It is apparent that for people with $\tau \in [0, \tau^*)$, nothing changes as effort levels have not changed. For people with $\tau \in [\tau^*, \tau_H^*)$, human capital declines. This is because these people no longer have an extra incentive to work hard just to get into college, leading to a reduction in human capital through the reduced effort as well as through losing out on a college education. For people with $\tau \in [\tau_H^*, \tau_0]$ human capital increases as a manifestation of the increased effort that they must now put in to secure a college seat. The same holds true for people with $\tau \in [\tau_0, \tau_{0H})$, for whom getting into college has become a binding constraint. Lastly, for people with $\tau \in [\tau_{0H}, \infty)$, human capital is unchanged as effort is unchanged.

The effect on average human capital, again, depends on $F_H(\tau)$, and if there is enough probability mass in $[\tau_H^*, \tau_{0H})$ then it is possible for high-caste average human capital for to increase as a result of affirmative action. □

Lemma 5.11. *For the low-caste, realized human capital rises only for those who have gained college seats due to affirmative action. For others, human capital either falls or stays constant.*

Proof. We know that $\tau_L^* \leq \tau^*$ and $\tau_{0L} \leq \tau_0$. We assume again that the two distributions of τ aren't very far apart so that $\tau^* \leq \tau_{0L}$. Analogous to Figure 5.9, we can then draw Figure 5.10.

The analysis proceeds as before. For people with $\tau \in [0, \tau_L^*)$, human capital is unchanged

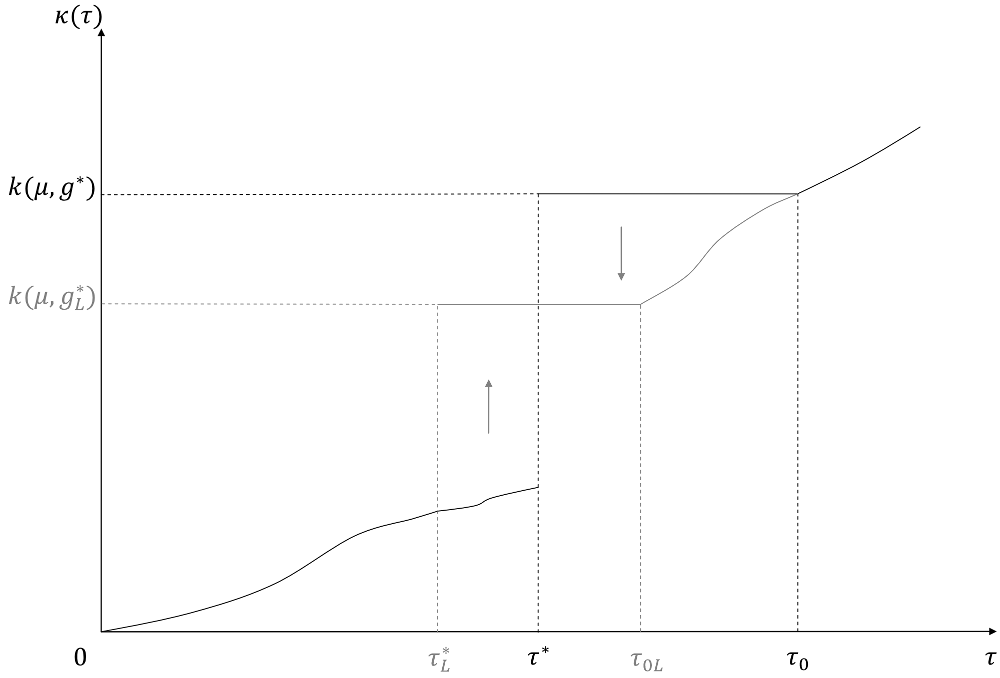


Figure 5.10: Effect on low-caste Human Capital

because effort is unchanged. People with $\tau \in [\tau_L^*, \tau^*)$ have a new incentive to work hard as it is now worthwhile for them to go to college. Human capital increases for them through the increased effort as well as through gaining a college education, which increases human capital for any effort level. For people with $\tau \in [\tau^*, \tau_{0L}]$ human capital falls because of the reduced effort they must put in to secure a college seat. For people with $\tau \in [\tau_{0L}, \tau_0)$ getting into college is no longer a binding constraint, and so effort levels are reduced, leading to a fall in human capital. Lastly, for people with $\tau \in [\tau_0, \infty)$, human capital remains constant as there are no incentives to change effort levels.

As above, the effect on average human capital depends on $F_L(\tau)$, and if there is enough probability mass in $[\tau^*, \tau_0)$ then it is possible for low-caste average human capital to decrease as a result of affirmative action. □

Theorem 5.6. *If enough probability mass of $F_H(\tau)$ lies in $[\tau_H^*, \tau_{0H})$ and enough probability mass of $F_L(\tau)$ lies in $[\tau^*, \tau_0)$, then inequality in human capital will increase.*

Proof. Follows immediately from Lemmas 5.10 and 5.11. □

As an illustration of such a situation, let us examine Figure 5.11. As we can clearly see, the increase in effort for the low-caste in the range $[\tau_L^*, \tau^*)$ will very likely be outweighed by the decline in the range $[\tau^*, \tau_0)$, so that average human capital falls. Similarly, for the high-

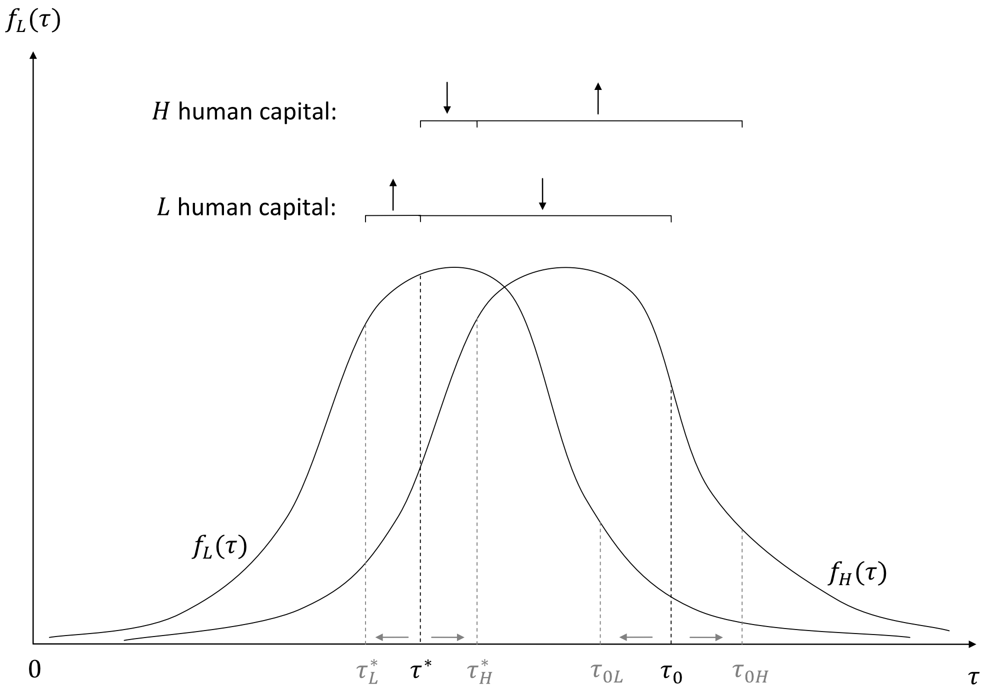


Figure 5.11: An Illustration of Increasing Inequality

caste, the reduction in effort in the range $[\tau^*, \tau_H^*)$ will likely be outweighed by the increase in the range $[\tau_H^*, \tau_{0H})$.

This diagram gives rise to a few rules of thumb. Firstly, notice that since the increase in high-caste human capital and the decrease in low-caste human capital come to the right of the initial cut-off, if the initial cut-off is towards the ends of the distributions (beyond which there is comparatively little mass), the low-caste decline and the high-caste increase is experienced by very few people, and inequality is more likely to decrease. If, on the other hand, the initial cut-off is at a point beyond which there is substantial mass in both distributions, then inequality is likely to widen. The position of the cut-off depends on the number of seats available relative to the population; a small number means a high cut-off, whereas a moderate number will mean a cut-off more towards the middle of the distributions. One must take care while thinking about this in terms of real world scenarios, though, as in our model, everyone in the population goes to school and would get a higher payoff by going to college if they did not have to compete for a limited number of places (i.e. if the *GG* curve didn't exist). In the real world, especially in developing countries, not everyone goes to school, even fewer finish, and not everyone who finishes would have a higher payoff from going to college in an unconstrained world (e.g. children who take up farming as an occupation might view going to college as more of an opportunity cost). Our 'population' should thus be compared to the real world set of people who finish school and would prefer to go to college in an unconstrained world.

Secondly, the distance between τ^* and τ_0 (and that between τ_H^* and τ_{0H}) is crucial, as a large distance means that an increase in inequality is likely. Intuitively, this distance will be large if a college education adds a lot to school education. This is because if there is more to be gained from a college education relative to a school education, more people are likely to find it attractive to work 'extra-hard' to get into college, and these are precisely the people who constitute the range $[\tau^*, \tau_0)$.

Lastly, the distance between τ^* and τ_i^* for each caste is also important, as small distances mean that an increase in inequality is likely. These distances are determined by how far apart the distributions are. If the distributions are not very far apart, the adjustments required to achieve equal representation are small, as there would not be a very large difference between the probability masses of the two distributions to the right of the initial cut-off. This provides an interesting opportunity for a dynamic model, as intuitively we could have a state of permanent inequality despite affirmative action. The reasoning would be that if the the distributions are far apart to start off with, other things staying constant, inequality would reduce for the current generation. But since human capital usually translates very well into income, the income of the next generation's parents would be more equal, and so their distributions of τ would also be more equal. But this means that the distance between τ^* and τ_i^* for the next generation will be smaller, leading to a smaller decrease in inequality. This process would continue until the point where the distributions of τ become close enough so that inequality no longer decreases. Indeed, one could have a case where affirmative action increases inequality over time if the distributions are sufficiently close together to begin with. Formalizing these ideas is a task for future research.

5.3.5 Effect on Students' Welfare

Lemma 5.12. *For the high-caste, welfare falls in the range (τ^*, τ_{0H}) and remains constant elsewhere. Average welfare falls.*

Proof. This situation is depicted in Figure 5.6 with $i = H$.

For people with $\tau < \tau^*$, we know that effort levels do not change and they still do not go to college. So, welfare remains unchanged. People with $\tau = \tau^*$ were earlier indifferent between going to college and not going. Now that going to college involves a higher effort cost than before, they strictly prefer not going to college, but since the payoff from not going to college hasn't changed, they continue to receive the same payoff. People with $\tau \in (\tau^*, \tau_H^*)$ have lost college seats due to affirmative action, and earlier strictly preferred going to college to not going. An increase in the effort levels required to go to college has reduced the payoff from going to college, so that they now prefer not to go. Since they strictly preferred to go earlier, this reduces welfare. People with $\tau \in [\tau_H^*, \tau_0)$ still go to college, but must now work even harder to do so, which clearly reduces welfare. For people with $\tau \in [\tau_0, \tau_{0H})$, going to college has become a binding constraint. They are forced to work harder in order to secure a college seat whereas earlier they operated on their unconstrained maxima. The move from unconstrained to constrained maxima implies a reduction in welfare. People with $\tau \geq \tau_{0H}$ continue to go to college without any increase in effort, and so welfare remains unchanged.

As welfare falls over some ranges and remains constant over others, average welfare for the high-caste must fall. □

Lemma 5.13. *For the low-caste, welfare increases in the range (τ_L^*, τ_0) and remains constant elsewhere. Average welfare rises.*

Proof. The effect on low-caste welfare is depicted in Figure 5.12.

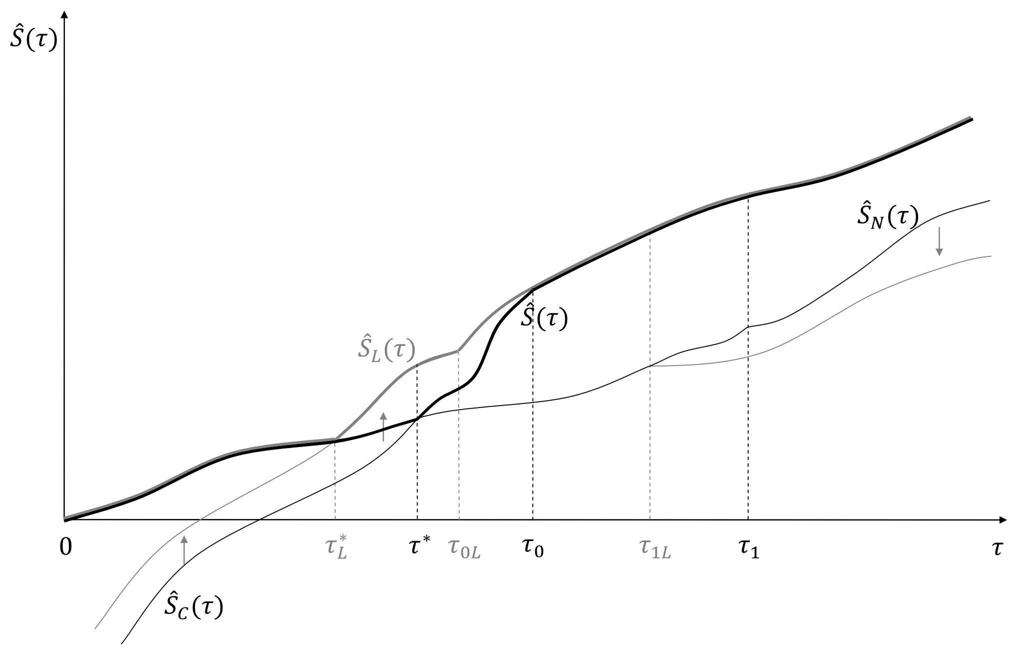


Figure 5.12: Effect on low-caste Welfare

People with $\tau < \tau_L^*$ do not go to college, with or without affirmative action. We also know their effort levels are unchanged, so their welfare in unchanged. People with $\tau = \tau^*$ earlier strictly preferred not going to college, but are now indifferent between going and not going. As the payoff from not going hasn't changed, the indifference implies that they receive the same payoff. For people with $\tau \in (\tau_L^*, \tau^*)$ have gained college seats. They earlier strictly preferred not going to college, but the reduction in cut-off has meant that getting into college requires less effort, and this has increased the payoff from going to college to an extent that they now prefer to go to college. This must increase welfare, as the payoff from not going to college has not changed. People with $\tau \in [\tau^*, \tau_{0L})$ continue to go to college, but have to put in less effort to secure a college seat. This moves them closer to their unconstrained optima, which, as we have earlier argued, must increase welfare. For people with $\tau \in [\tau_{0L}, \tau_0)$, getting into college is no longer a constraint, and they can continue to go to college while reducing their effort levels to the unconstrained optima, which must give them higher welfare than operating at constrained optima. Lastly, people with $\tau \geq \tau_0$ continue to go to college and operate on their unconstrained optima, so welfare remains constant.

Average welfare must rise, as over some ranges of τ welfare rises while over others it remains constant. □

5.4 Summary of Main Results

In this chapter, we introduced students as economic decision makers by allowing them to choose an effort level in order to maximize their human capital. We noted an important discontinuity in the students' payoff which arose from the value added by college for those people who managed to secure a college seat. We found that this discontinuity had interesting implications for effort incentives. Students who were not very well equipped to gain from education settled for a school education and chose low effort levels. Students who were very well equipped chose moderately high levels of effort, and got into college without trying 'extra-hard'. However, students who were moderately equipped faced incentives to work 'extra-hard' to secure admission into college. We also found that realized human capital and welfare were unequally distributed.

Affirmative action had interesting effects on effort. While effort levels increased for those low-caste students who gained college seats, effort incentives were reduced for the low-caste candidates who earlier had to work 'extra-hard' to get into college as the reduction in cut-off meant it took less work to secure admission. For the high-caste, the effect was exactly the opposite, with reduced effort for those losing college seats, but increased effort for those who still went to college, but had to work even harder to achieve the increased cut-off grade. We found that these effects were transferred to human capital, and this gave rise to the possibility of increased inequality as a result of affirmative action under certain conditions. Welfare, however, unambiguously fell for the high-caste and rose for the low-caste.

Chapter 6

Conclusion and Avenues for Further Research

This thesis is among the first contributions that seek to provide a theoretical framework for the analysis of affirmative action policies in countries with institutional structures different from the US. Specifically, college admissions are not taken to be decisions made by economic agents, rather they are completely mechanical.

In the baseline model, we found that inequality in human capital decreases due to a shift in college seats from high to low-caste candidates. However, there is a fall in economy-wide human capital, because the gainers are less able to take advantage of a college education by virtue of being less qualified than the high-caste candidates they displace.

In the differences in school quality model, we found that good quality schools catering to the high-caste experience a reduction in quality because the relative attractiveness of low-caste students increases leading to reduced incentives to bid up quality to attract high-caste students. Bad quality schools face an incentive to shift quality in a way that puts more people in the vicinity of the cut-off grade, so as to take full advantage of the reduction in the cut-off. There is thus the possibility that bad quality schools also experience a quality reduction, leading to a general reduction in school quality and in extreme cases a general reduction in human capital without any beneficial effect on inequality.

In the effort choice model, we found that while affirmative action does increase effort incentives for the section of the low-caste population that acquires college seats, for other low-caste people, effort incentives are reduced as ensuring a college seat becomes much easier. Similarly, although affirmative action reduces effort levels for the high-caste people that lose seats, it increases effort incentives for other high-caste members as ensuring a college seat has become much tougher. This can again give rise to perverse effects because if effort increases for a sufficient number of high-caste people and reduces for a sufficient number of low-caste people, human capital attainment may increase for the high-caste and decrease for the low, leading to an increase in inequality.

Affirmative action is a hotly debated topic the world over, and this thesis has been an attempt to more rigorously think about and understand some of the incentive structures at play. The value of such work to the policy debate is obvious.

This thesis is by no means a completely rigorous analysis of the topic. Many issues remain unaddressed. This research could be further enriched and complemented by future research along the following lines.

1. **Dynamics.** A desirable addition to this method of analysis would be an explicit inter-generational inequality transmission mechanism that preserves the basic features of these models. Perhaps one could have the human capital of one generation translate into the parental income for the next. This will allow us to investigate whether, given initial taste-based discrimination, inequality will ever fade away, and if so, how quickly. We could then investigate whether the introduction of affirmative action can break states of perpetual inequality, or indeed lead to the reduction of inequality at a faster rate. In Chapter 5 we argued that there may be cases where affirmative action itself could lead to perpetual inequality.

2. **Empirical estimation and validation.** The usefulness of this research to policy makers would be magnified if data that allows the estimation of the joint distribution of ability and parental income, the school and college human capital production functions and associated cost functions could be collected. The models could be re-formulated as structural models, and the estimated parameters would allow more specific predictions about the effects of affirmative action policies under prevailing conditions. This thesis has noted that affirmative action may have perverse effects under certain conditions; empirical work is required to test whether those conditions hold in reality. Also, once the prevailing conditions are identified, data on incomes could be used to validate the results.

3. **Liquidity constraints and costly education.** We have abstracted so far from the

obvious reality that often good quality schools charge high fees, which cannot be afforded by everyone. Access to education may thus be restricted by the ability to pay, and if this ability is unevenly distributed, then liquidity constraints alone could result in inequality being perpetuated.

4. **Peer effects.** We have throughout this thesis we have not considered whether a student's human capital formation is influenced by the education level of his peers. Other authors[1] have considered such externality effects important. Research needs to be done into whether these effects exist in practice and, if so, these effects must be incorporated into the models presented in the thesis.

5. **Strength of affirmative action policies.** In this thesis, we have only considered affirmative action policies that aim at equal representation at the college level. We could extend this work by letting the strength of affirmative action policies vary, and examine outcomes in different scenarios, analogous to the work in Fryer et al. (2008). We may also build a model of the optimal strength of affirmative action policies.

6. **Alternative policies for reducing inter-caste inequality.** Serious thought needs to be given to alternate methods of reducing inequality, and their functioning must be compared to affirmative action as it exists today. For instance, could policies based on income targeting rather than caste targeting better achieve the desired results? In addition, should primary and secondary education be the focus rather than higher education?

[1] For example see Durlauf (2008) and De Fraja (2002).

References

Bertrand, M., Hanna, R., & Mullainathan, S. (2008, April). *Affirmative action in education: Evidence from engineering college admissions in India* (Working Paper No. 13926). National Bureau of Economic Research. Available from http://www.nber.org/papers/w13926

Coate, S., & Loury, G. C. (1993). Will affirmative-action policies eliminate negative stereotypes? *The American Economic Review*, *83*(5), 1220–1240.

De Fraja, G. (2002, May). *Affirmative action and efficiency in education* (Working Paper No. 3357). Centre for Economic Policy Research.

Desai, S., & Kulkarni, V. (2008). Changing educational inequalities in India in the context of affirmative action. *Demography*, *45*(2), 245–270.

Durlauf, S. N. (2008). Affirmative action, meritocracy, and efficiency. *Politics Philosophy Economics*, *7*(2), 131–158.

Epple, D., Romano, R., & Sieg, H. (2008, August). Diversity and affirmative action in higher education. *Journal of Public Economic Theory*, *10*(4), 475–501.

Fernández, R., & Rogerson, R. (2001). Sorting and long-run inequality. *The Quarterly Journal of Economics*, *116*(4), 1305–1341. Available from http://www.jstor.org/stable/2696460

Fryer, R. G., Jr, Loury, G. C., & Yuret, T. (2008, October). An economic analysis of color-blind affirmative action. *Journal of Law, Economics, and Organization*, *24*(2), 319–55.

Ghosh, J. (2006, June). Case for caste-based quotas in higher education. *Economic and Political Weekly*, *41*(17), 2428–2432.

Howell, J. S. (2004). *A structural equilibrium model of the market for higher education: Assessing the impact of eliminating affirmative action.* Unpublished doctoral dissertation, University of Virginia.

Kingdon, G. (1996, February). The quality and efficiency of private and public education: A case-study of urban India. *Oxford Bulletin of Economics and Statistics*, *58*(1), 57–82.

Milgrom, P., & Shannon, C. (1994). Monotone comparative statics. *Econometrica*, *62*(1), 157–180.

Monks, J., & Ehrenberg, R. G. (1999, July). *The impact of US News and World Report college rankings on admission outcomes and pricing decisions at selective private institutions* (Working Paper No. 7227). National Bureau of Economic Research. Available from `http://www.nber.org/papers/w7227`

Nechyba, T. J. (2003). Centralization, fiscal federalism, and private school attendance. *International Economic Review*, *44*(1).

Rey, E. D., & Racionero, M. (2008). An efficiency argument for affirmative action in higher education. *Hacienda Pública Española*, *187*, 41–48.

Rotthoff, K. W. (2008). Could affirmative action be efficient in higher education? *Economics Letters*, *99*(3), 574–576.